

A MATHEMATICAL MODEL AND SOLUTION APPROACH FOR A HETEROGENEOUS FLEET OPEN VEHICLE ROUTING PROBLEM

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ABSTRACT: This research proposes a new mathematical model and a solution method for a heterogeneous fleet open vehicle routing problem (HFOVRP). HFOVRP focuses on determining the set of customers and delivery routes for a mixed fleet of vehicles that does not return to the depot after servicing the last customer on a route. This situation usually occurs in manufacturing companies or warehouses that use outsource delivery companies, as well as in delivery companies that use third party logistics companies. In this research, HFOVRP is applied at a government enterprise that hires outsource delivery companies to distribute parts from the depot to 49 regional warehouses. At present, the company uses a point-to-point delivery system. Therefore, the company tries to collect orders from each destination warehouse until there is a full truckload or the lead time to reduce the transportation cost is met. However, a full truckload may not be achieved when the destination warehouse has to wait for parts for a very long time. This research proposes a new delivery system that collects the orders from every destination warehouse and creates a cluster of destination warehouses and optimal vehicle routes, which is considered as a HFOVRP. The mathematical model is formulated by employing a modified heterogeneous fleet vehicle routing problem. The solution algorithm starts with clustering the warehouses, dealing with overload demand, and solving problems by using a branch-and-cut algorithm. After performing the numerical experiments, the results show that the transportation cost was reduced by 15.02%.

Keywords: Mathematical model, Heterogeneous fleet, Single depot, Open vehicle routing problem

1. INTRODUCTION

Transportation and distribution management has a crucial role in logistics management to enhance customer satisfaction and the company's core competency. The efficient distribution of products to customers at the lowest cost is one of the most challenging problems in logistics management. This problem involves finding the optimal assignment of customers to be visited by a fleet of vehicles, selecting the vehicle routes, and calculating the number of vehicles needed to serve the customers. This type of problem is called the Vehicle Routing Problem (VRP).

VRP has been extensively studied since it was first introduced by [1] in 1959. The current VRP models are very different from the first model as they increasingly aim to incorporate real-life complexities, such as time-dependent travel time, time windows for pickup and delivery, depot visiting after servicing the last customer and input information that changes over time [2]. The open vehicle routing problem (OVRP) is well-known variant of VRP. The problem is first described in the literature by [3] in 1981 but has only recently attracted the attention of scientists and researchers [4]. It differs from the classical VRP in that a fleet of vehicles does not necessarily return to their original location after serving customers; if they do,

they must follow the same path in the reverse order [5]. There are many applications in industry and service for businesses that do not usually have fleets [6]. These companies use outsource vehicles through third party logistic companies. The classical VRP and OVRP can be depicted in Fig. 1 and Fig.2 respectively.

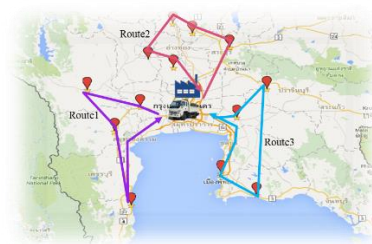


Fig.1 The classical vehicle routing problem

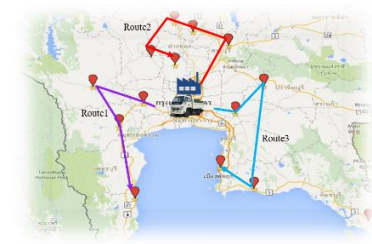


Fig.2 The open vehicle routing problem

In real-life routing applications, mixed fleets with variant capacity are common. When this condition is considered in VRP, it is known as the Heterogeneous Fleet Vehicle Routing Problem (HFVRP) or the mixed fleet vehicle routing problem [7]. In this paper, open routes are investigated in HFVRP. Consequently, the problem is a Heterogeneous Fleet Open Vehicle Routing Problem (HFOVRP). HFOVRP finds the optimal number of vehicles and designs optimal delivery routes to be used by a mixed fleet of vehicles to serve a set of customers at minimum delivery cost subject to the vehicle capacity constraint. The fleet of vehicles starts at a central depot and is characterized by different capacities and costs. After serving the last customer on the route, the vehicles do not return the depot. According to the literature review, there have been a few similar studies on the open routing version of HFVRP with a variant including constraints, such as [8] - [9]. Therefore, to explore the algorithms for HFOVRP, these articles and other variant versions of OVRP are studied.

OVRP is an NP-hard problem. Most mathematical models for the variant class of OVRP are constructed by modifying the class of VRP. Most of them are integer programming problems. Classical exact algorithms such as the branch-and-cut algorithm [10], column generation [9], etc., are only efficient in instances of small problems. Therefore, most research focuses on heuristics and metaheuristics such as the ant colony algorithm [6], the hybrid genetic algorithm [8], tabu search [11], the particle swarm algorithm [12], the hybrid algorithm [13], and so on.

Even though an exact algorithm takes more computational time than heuristic and metaheuristic algorithms, it guarantees an optimal solution. Moreover, a significant increase in the performance of computers due to new technology and the parallel computing technique dramatically reduces the limitation of problem size when solved by an exact algorithm. There is much software supporting exact algorithms. Most are commercial software but some are freeware such as OpenSolver, etc.

In this paper, the case study company is a government organization that is responsible for the provision of standardized electricity services and related business. To meet that mission, the warehouse management division at the case study organization needs to distribute supplies from its depot to 49 regional warehouses. At present, point-to-point delivery is carried out to all warehouses. The supply orders from each destination warehouse are collected until there is a full truckload or the delivery deadline is reached, whichever occurs first. These supplies are delivered by an outsource logistics company using two types of vehicle, which are 10-wheel trucks and trailer trucks. The types of

vehicle are selected to match the quantity of delivered supplies. In each delivery round, each vehicle starts by loading supplies at the main warehouse (hereafter referred to as the depot), and then travels to only one destination warehouse without returning to the depot. The company is aware of the lateness and the high delivery cost of using the present delivery method and targets cost reductions. Therefore, the organization needs to improve the delivery policy by grouping the destination warehouses and by creating optimal vehicle routes to reduce transportation cost by at least 10%. Therefore, we propose HFOVRP instead of the current delivery system. The mathematical model of HFOVRP is obtained by modifying the HFVRP model proposed in [14]. At first, the customers are grouped by using the angular sweep algorithm. The branch-and-cut algorithm is then employed to solve the model in each cluster. The program using VBA and OpenSolver in Excel is constructed to support delivery planning.

The remainder of this paper is organized as follows. In the next section, the research methodology is presented. After that, an example of program implementation and the numerical results are shown. Finally, the conclusion and discussion on the results are presented.

2. RESEARCH METHODOLOGY

The research methodology has five parts: studying the current delivery problem and restrictions, formulating a mathematical model, developing the solution algorithm, developing a delivery planning program, and conducting numerical experiments.

2.1 Problem Description and Restrictions

The proposed delivery system, HFOVRP, changes the vehicle management of the current delivery system, which is point-to-point delivery, without changing the following conditions and restrictions. The organization outsources deliveries from the depot to 49 regional warehouses. The working hour limit of the driver is set at 8 hours. Nine out of these 49 warehouses require nearly 8 hours of travel time, so they cannot be delivered together with the others. Therefore, point-to-point delivery is still needed for these nine warehouses and the remaining 40 warehouses are considered in the study. The distance between each pair of warehouses and the distance between each warehouse and the depot are estimated with a geometric information system (GIS). The delivery cost includes driver wages and fuel cost, which depends on the type of vehicle and the distance.

Two types of vehicles, which are 10-wheel trucks and trailer trucks, have limited capacity at 24,000 units and 12,000 units respectively. The number and type of vehicles need to be decided. The maximum number of available vehicles is unlimited. All vehicles start at the depot, but they do not return to the depot. The traveling time, which ignores traffic conditions, is obtained by dividing the distance by the speed limit. Each warehouse is served by only one vehicle. The maximum order from any warehouse does not exceed the capacity of the available vehicles. The required space for all items is converted into the same unit of measurement.

2.2 Mathematical Model Formulation

The mathematical model representing the case study organization delivery problem is developed by modifying the mathematical model for HFVRP proposed by [14]. The travel cost of all returning paths to the depot are deleted from the objective function and the traveling time of these paths is not considered in the working hour constraint. The notations used in the formula are as follows. The vertex set $V = \{0, 1, \dots, N\}$ is the set of $N+1$ nodes. Node 0 represents the factory, while the remaining node set $V' = V \setminus \{0\}$ corresponds to N customers. $K = \{1, \dots, M\}$ is the vehicle set. The vehicles are indexed by parameter k running from 1 to M and each type of vehicle has a different capacity (Q_k). M , which is total number of vehicles, is unknown at the first stage. The maximum number of 10-wheel trucks (M_1) and the maximum number of trailer trucks (M_2) are independently calculated by dividing the total quantity of demand by the capacity of the vehicles, which is used as the upper bound. The number of vehicles used can be less than or equal to this upper bound. M is the summation of M_1 and M_2 . Indexes of $k=1$ to M_1 are used for 10-wheel trucks and $k=M_1+1$ to M are used for trailer trucks. Q_1 to Q_{M_1} equals 12,000 units and Q_{M_1+1} to Q_{M_2} equals 24,000 units. c_{ijk} is the delivery cost, which is the transportation charge, from node i to node j of vehicle k . The transportation charge consists of $c_{iik} = \infty$. Parameter q_i is the number of units of product at node i . t_{ijk} is traveling time from node i to node j by vehicle k . s_j is the service time at node j . The decision variables and the mathematical model of this problem are as follows.

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{if node } i \text{ is visited by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

u_{ik} = arbitrary real numbers which satisfy constraints (7)

$$\text{Minimize } Z = \sum_{k \in K} \sum_{i \in V} \sum_{j \in V'} c_{ijk} x_{ijk} \quad (1)$$

subject to

$$\sum_{i \in V'} q_i y_{ik} \leq Q_k \quad \forall k \in K \quad (2)$$

$$\sum_{k \in K} \sum_{i \in V} x_{ijk} = 1 \quad \forall j \in V' \quad (3)$$

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \quad \forall i \in V' \quad (4)$$

$$\sum_{i \in V} x_{ipk} - \sum_{j \in V} x_{pj k} = 0 \quad \forall p \in V', \forall k \in K \quad (5)$$

$$u_{ik} - u_{jk} + n x_{ijk} \leq n - 1 \quad \forall (i, j) \in V', i \neq j, \forall k \in K \quad (6)$$

$$\sum_{j \in V'} x_{0jk} \geq \sum_{j \in V} x_{ijk} \quad \forall i \in V', \forall k \in K \quad (7)$$

$$\sum_{j \in V'} x_{0jk} \leq 1 \quad \forall k \in K \quad (8)$$

$$\sum_{j \in V'} \sum_{i \in V} x_{ijk} (t_{ijk} + s_j) \leq L \quad \forall k \in K \quad (9)$$

$$x_{ijk} \in \{0, 1\} \quad \forall (i, j) \in V, \forall k \in M \quad (10)$$

$$y_{ik} \in \{0, 1\} \quad \forall i \in V, \forall k \in M \quad (11)$$

The objective function (1) states that the total travel cost, excluding the cost of returning to the depot, is to be minimized. Constraint (2) ensures that the total quantity of product loaded on a vehicle is not over the truckload. Constraint (3) and (4) ensure that each customer is visited by one vehicle. Route continuity is represented by constraint (5), i.e., if a vehicle enters a demand node, it must exit from that node. Constraint (6) is the subtour-breaking constraint. Constraints (7) - (8) state that if a vehicle travels to any customer, then that vehicle must leave from the depot to only one customer. Constraint (9) states that the total time (travel time and service time) spent by each vehicle must not be over the daily working hour limit. The travel time to return to the depot is excluded in this constraint. Constraints (10) - (11) are logical constraints.

2.3 Solution Algorithm

The algorithm is composed of three main steps: clustering the warehouses, dealing with the overload demand, and solving the vehicle routes. The first two steps are pre-process phases for the third step. They reduce the problem size using the transportation constraints to enhance the efficiency of the solution algorithm. These steps are described as follows.

2.3.1 Clustering the warehouse

In this step, the warehouses are clustered according to the limitation of daily working hours (8 hours). Regarding the traveling time and service time data, no more than 4 warehouses can be served by a single vehicle. Therefore, 40 warehouses are equally split into 10 clusters using the angular sweep algorithm. These clusters are shown in Table 1.

Table 1 Warehouses in each cluster

BRANCH NAME	BRANCH NAME
Cluster 1	Cluster 6
1 HuaHin	1 Phra Phutthabat
2 Pethaburi	2 Lopburi
3 Samut Sakhon	3 Tha Ruea
4 Samut Songkhram	4 Khok Samrong
Cluster 2	Cluster 7
1 Samphran	1 Kabinburi
2 Nakhonchaisri	2 Saraburi
3 Ratchaburi	3 Nakhonnayok
4 Photharam	4 Prachinburi
Cluster 3	Cluster 8
1 Suphanburi	1 Thanyaburi
2 Banpong	2 Chachoengsao
3 Kanchanaburi	3 Srakaew
4 Ladlumkaew	4 Panat Nikhom
Cluster 4	Cluster 9
1 Sena	1 Rayong
2 Ladyao	2 Bang Pakong
3 Uthaithani	3 Klaeng
4 Chainat	4 Pluak Daeng
Cluster 5	Cluster 10
1 Ang Thong	1 Sriracha
2 Ayuthaya	2 Chonburi
3 Singburi	3 Pattaya
4 Nakhonsawan	4 Rangsit

2.3.2 Dealing with overload demand

In cases where the quantity of supplies from a warehouse is over the capacity of the trailer truck, the order is divided into two parts. The quantity that can fit in the trailer trucks is assigned to trailer trucks. Point-to-point delivery will apply for the full trailer loads. The capacity of a trailer truck is twice that of a 10-wheel truck but the unit travel cost is less than twice. Using one trailer truck is cheaper than using two 10-wheel trucks. The remaining quantity of supplies is considered in the next step of the algorithm.

2.3.3 Solving the vehicle routes

Having completed the previous two steps, the problem size is thus reduced. The mathematical model, which is an integer programming problem, is solved. To solve the problem, the branch-and-cut

algorithm in OpenSolver is employed because of the free software license and user familiarity.

2.4 Delivery Planning Program Development

To facilitate logistics planners, the delivery planning program is developed using Excel VBA. This program is divided into three parts: the data input part, the solver part, and the report part.

The data input spreadsheet shown in Fig. 3 is created to input the orders of each warehouse. The “Reset” button allows users to clear the past data in this spreadsheet. After the operators click the “Solve” button, the second part of the program, the solver spreadsheet, will be activated. The input demands that are over the trailer truck capacity (24,000 units) are subtracted from the trailer truck capacity and kept in the report spreadsheet. These orders will be delivered point-to-point. The remaining orders will be sent to the solver part.

The solver part consists of 10 spreadsheets corresponding to 10 clusters to respond for the case of a specific cluster is of interest. For example, Fig. 4 and Fig. 5 show the imported data in spreadsheet of cluster 4 and solver in spreadsheet of cluster 4 respectively.

BRANCH NAME	Demand (Units)
Cluster 1	
1 HuaHin	
2 Pethaburi	
3 Samut Sakhon	
4 Samut Songkhram	
Cluster 2	
1 Samphran	
2 Nakhonchaisri	
3 Ratchaburi	
4 Photharam	
Cluster 3	
1 Suphanburi	23500
2 Banpong	6000
3 Kanchanaburi	
4 Ladlumkaew	5000
Cluster 4	
1 Sena	36000
2 Ladyao	
3 Uthaithani	12000
4 Chainat	12000
Cluster 5	
1 Ang Thong	6000
2 Ayuthaya	12000
3 Singburi	6000
4 Nakhonsawan	
Cluster 6	
1 Phra Phutthabat	
2 Lopburi	5970
3 Tha Ruea	
4 Khok Samrong	18000
Cluster 7	
1 Kabinburi	
2 Saraburi	36000
3 Nakhonnayok	6000
4 Prachinburi	4000
Cluster 8	
1 Thanyaburi	
2 Chachoengsao	16500
3 Srakaew	
4 Panat Nikhom	
Cluster 9	
1 Rayong	33030
2 Bang Pakong	5000
3 Klaeng	
4 Pluak Daeng	12000
Cluster 10	
1 Sriracha	
2 Chonburi	12000
3 Pataya	
4 Rangsit	12000

Reset

Solve

Fig. 3 Data input spreadsheet

80,751.204 Baht. Point-to-point delivery is carried out to all warehouses. Each vehicle starts by loading supplies at the depot, and then travels to only one assigned destination warehouse without returning to the depot. Since the warehouses cannot share the space of vehicles, the number of vehicles is higher than that of the proposed delivery system.

After implementing the program with 30 real data sets, the program yields delivery plans within 5 minutes. The transportation cost of these data sets is illustrated in Fig. 7. The transportation cost of the proposed delivery system is lower than the transportation cost of the original delivery system for all data sets. According to the numerical results, the delivery cost is reduced by 15.02% on average.



Fig.7 Comparison of transportation cost

4. CONCLUSION

In this research, a delivery planning program is developed to determine vehicle usage and to provide an optimal delivery route for a fleet of vehicles in each cluster of customers of the case study company. After program trial, the program allows the planners to make better decisions on delivery planning, the delivery cost is decreased by 15.02% on average. The new mathematical model applied in the program can represent real problems. The branch-and-cut algorithm in OpenSolver can solve the model, which is a MIP, in an acceptable computing time.

5. RECOMMENDATIONS

For the future research, there are two main extensions of this research. The first area is to modify the algorithm to find a tighter upper bound for the number of 10-wheel trucks and trailer trucks to reduce the computational time. In this paper, M1 and M2 are calculated independently, then M is much larger than the number of vehicles need. Therefore, the problem involves a lot of decision

variables. The efficient heuristic algorithm is another alternative. The second extended version aims to expand the scope and enhance the realism of the mathematical model by considering conditions such as longer planning period, traffic conditions, split load and so on.

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