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A NEW ESTIMATION PROCEDURE USING A REVERSIBLE JUMP MCMC ALGORITHM FOR AR MODELS OF EXPONENTIAL WHITE NOISE

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ABSTRACT: The autoregressive model generally has a Gaussian error. If an autoregressive model that has a Gaussian error is used to model data, the assumption of normality is often not obeyed by the data. In addition, the parameters of the autoregressive model are generally unknown. The parameters of the autoregressive model include order model, model coefficient, error mean and error variance. This paper aims to determine the parameter estimation procedure of an autoregressive model that has an exponential error. In this paper, the autoregressive model parameter estimation is worked out in a hierarchical Bayesian framework. Since the autoregressive order is also part of the model parameter, the Bayes estimator has a complex form so that the Bayes estimator cannot be explicitly calculated. To solve the problem, the reversible jump MCMC is implemented. The results show that model order, model coefficient, error mean and error variance can be calculated simultaneously. In addition, the resulting autoregressive model is always a stationary autoregressive model.

Keywords: Exponential Error, Autoregressive Model, Hierarchical Bayesian, Reversible Jump MCMC

1. INTRODUCTION

An autoregressive (AR) model with normally distributed white noise is a time series model that is often used in many fields. For example, it is used in the field of economics [1]. But there are so many applications where white noise is not normally distributed. An LSE of AR models with heavy-tailed G-GARCH(1,1) noises was studied [1]. A class of nonparametric tests on the Pareto tail index of the innovation distribution in the linear autoregressive model is proposed [2]. A study of the autoregressive models with exponential white noise can be found in the literature [3]-[8]. A form of time series models where marginal distributions are in fact exponential distributions is presented in [3]. A Bayesian analysis of threshold AR models with exponential noise is developed in [4]. A robust study of the Bayesian estimation of an AR model with exponential innovation to obtain optimal Bayesian estimator is analyzed [5]. A Bayesian method to estimate the coefficient of the AR(1) models are proposed [6]. Generally, the order of the autoregressive is known and must be estimated from the data.

If the AR model with white noise is fitted to the data, the order and the coefficient of the model will be generally unknown. Let x_t be a time series with $t = 1,2,\ldots,n$ and n be the number of samples. An AR(p) with exponential white noise can be expressed as:

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + z_t \tag{1}$$

where $p < n$ and the $z_t\,(t = 1,\ldots,n)$ are independent and identical exponential random variables with a parameter λ, written $z_t \sim \mathrm{Exp}(\lambda)$. For example, Fig. 1 shows the graph of the autoregressive model with and $\lambda = 5$.

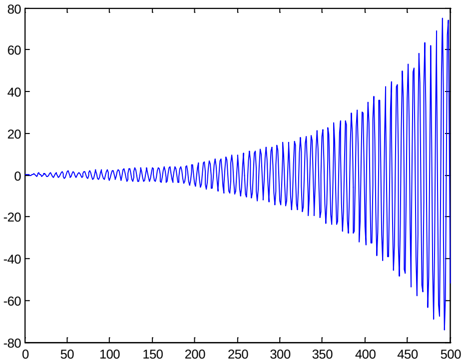

Fig. 1 AR(3) with exponential white noise

Suppose that $\phi^{(p)} = (\phi_1,\ldots,\phi_p)$ is a coefficient vector. Let ψ be the above autoregressive model. Then this parameter ψ can be written as

$$\psi = (p, \phi^{(p)}, \lambda)$$

Suppose that x_t ($t = 1,2,\ldots,n$) are data. This data is taken from a population having an autoregressive model with exponential white noise. Based on this data, the main problem becomes how to estimate the parameter ψ. This paper aims to provide a procedure to estimate the parameter ψ.

2. METHOD

The parameter ψ is estimated by using a Bayesian method. Unfortunately, the Bayesian estimator cannot be determined analytically because the posterior distribution of parameter ψ has a complicated form. To overcome these problems, a reversible jump MCMC Algorithm [9] is used.

Let E denote the set of states and π denote the probability of state on E. The Metropolis-Hastings algorithm produces the Markov chain on E which has stationary probability π. The Markov chain formation is based on reversibility conditions. The probability π is called stationary if for the kernel K of the Markov chain on E verifies:

$$\pi(x) = \sum_{y \in E} \pi(y) K(y,x)$$

for all $x \in E$. The probability π is called reversible for the kernel K if

$$\pi(x)K(x,y) = \pi(y)K(y,x)$$

for all $x, y \in E$. It is clear that the reversibility of π implies on the stationarity for kernel K. This property is used to form the kernel K such that π is a stationary distribution. Let q state the auxiliary kernel on E. Starting from $x \in E$, the withdrawal of a new point y is done in 2 stages:

- The point y is drawn according to q(x,y)
- The y point is accepted with probability

$$\rho(x,y) = \min\left\{1, \frac{\pi(y)q(y,x)}{\pi(x)q(x,y)}\right\}$$

Kernel K is defined as

$$\begin{cases} K(x,y) = q(x,y)\rho(x,y) & \text{if } x \neq y \\ K(x,y) = q(x,x) + \sum_{y \neq x} [1 - \rho(x,y)] & \end{cases}$$

The kernel K verifies the reversible equation. If the Markov chain is irreductible and aperiodic, the probability π is also the limit probability.

Let E denote the space formed by two different dimension spaces

$$E = \{1\} x \Re^{n_1} \cup \{2\} x \Re^{n_2}.$$

Here n_1 and n_2 are different integers. Then $\{1\}x\Re^{n_1}$ is written by $\Re^{n_1}$ and $\{2\}x\Re^{n_2}$ is written by $\Re^{n_2}$. The set E is formed by two elements i.e. one is an element of $\Re^{n_1}$ and one is an element of $\Re^{n_2}$. Similarly, the measure of π is formed by π_1 that is carried by $\Re^{n_1}$ and π_2 that is carried by $\Re^{n_2}$.

In $\Re^{n_1}$ or $\Re^{n_2}$, the Metropolis-Hastings algorithm can function without difficulty. Instead, it is necessary to define the transformation from $\Re^{n_1}$ to $\Re^{n_2}$ or from $\Re^{n_2}$ to $\Re^{n_1}$ that satisfy the reversible equation. Let q state the auxiliary kernel and ρ state the probability of acceptance/rejection. Then it must satisfy

$$\int_A \pi(dx) \int_B q(x,dx')\, \rho(x,x') = \int_B \pi(dx') \int_A q(x',dx)\, \rho(x',x)$$

for all $A \subset B_1$ and $B \subset B_2$. Or

$$\int_A \pi(dx) \int_B q_{12}(x,dx')\, \rho(x,x') = \int_B \pi(dx') \int_A q_{21}(x',dx)\, \rho(x',x)$$

Here q_{12} denotes the probability kernel from $\Re^{n_1}$ to $\Re^{n_2}$ and q_{12} denotes the probability kernel from within.

Suppose that measure and kernel have density function to Lebesgue measure

$$\iint_{AxB} \pi_1(x)\, q_{12}(x,x')\, \rho(x,x')\, dxdx' = \iint_{BxA} \pi_2(x')\, q_{21}(x',x)\, \rho(x',x)\, dx'dx = \iint_{AxB} \pi_2(x')\, q_{21}(x',x)\, \rho(x',x)\, dxdx'$$

or

$$\pi_1(x)\, q_{12}(x,x')\, \rho(x,x') = \pi_2(x')\, q_{21}(x',x)\, \rho(x',x).$$

So

$$\rho(x',x) = \min\left\{1, \frac{\pi_2(x')q_{21}(x',x)}{\pi_1(x)q_{12}(x,x')}\right\}$$

Then a measure ξ is formed on $E \times E$ which is symmetric and such that $\pi(dx)\, q(x,dx')$ has density function $f(x,x')$ and $\pi(dx')\, q(x',dx)$ has density function too. Recall that the measure of ξ is symmetric if and only if for all positive measured functions $\varphi(x,y)$ on $E \times E$ satisfy

$$\iint_{ExE} \varphi(x,y)\xi(dx,dy) = \iint_{ExE} \varphi(y,x)\xi(dx,dy)$$

Since this measure is symmetric, then

$$\int_A \pi(dx) \int_B q(x,dx')\rho(x,x') = \iint_{AxB} f(x.x')\rho(x,x')\xi(dx,dx') = \iint_{BxA} f(x'.x)\rho(x',x)\xi(dx,dx')$$

$$\int_B \pi(dx') \int_A q(x',dx)\rho(x',x)$$

$$= \iint_{BxA} f(x'.x)\rho(x',x)\xi(dx',dx)$$
$$= \iint_{BxA} f(x.x')\rho(x,x')\xi(dx,dx')$$

The reversible equation becomes

$$\iint_{BxA} f(x'.x)\rho(x',x)\xi(dx,dx')$$
$$= \iint_{BxA} f(x.x')\rho(x,x')\xi(dx,dx')$$

In order for this equation to be verified, it simply verifies

$$f(x',x)\ \rho(x',x) = f(x,x')\ \rho(x,x')$$

for all $(x,x') \in ExE$. Or

$$\rho(x,x') = \min\left\{1, \frac{f(x',x)}{f(x,x')}\right\}.$$

The next problem is how to form a symmetrical mesure on $E \times E$ and the density function f associated with the transformation.

2.1 Mesure

The general idea is to equip two spaces $\Re^{n_1}$ and $\Re^{n_2}$ to be in the same dimensional space. Suppose m$_1$ and m$_2$ are two positive numbers such that

$$n_1 + m_1 = n_2 + m_2$$

Next, the corresponding transformations are defined.

$$\begin{cases} g_2: & \Re^{n_1+m_1} & \to & \Re^{n_2} \\ & (x,x_1) & \to & g_2(x,x_1) \end{cases}$$

and

$$\begin{cases} g_1: & \Re^{n_2+m_2} & \to & \Re^{n_1} \\ & (x',x_2) & \to & g_1(x',x_2) \end{cases}$$

Assume that there is an injective of the transformations of the component, i.e, for i = 1, 2 satisfy

$$g_i(u,\alpha) = g_i(u,\beta) \Rightarrow \alpha = \beta$$

Assume also that there is an inversion formula that allows going backward. For all $x \in \Re^{n_1}$ and $x_1 \in \Re^{m_1}$, there is a single $x_2 \in \Re^{m_2}$ such that $g_1[g_2(x,x_1),x_2] = x$. Also, define a function h$_2$ from $\Re^{n_1}x\Re^{m_1}$ into $\Re^{m_2}$ by writing $x_2 = h_2(x,x_1)$ that satisfies the previous equation.

Symmetrically, for all $x' \in \Re^{n_2}$ and $x_2 \in \Re^{m_2}$ there is a single $x_1 \in \Re^{m_1}$ such that $g_2[g_1(x',x_2),x_1] = x'$. Define the function h$_1$ from $\Re^{n_2}x\Re^{m_2}$ into $\Re^{m_1}$ by writing $x_1 = h_1(x',x_2)$. Finally, the inversion properties are possible based on g$_1$ and g$_2$, to make two mutually inverse applications

$$\begin{array}{llll} \Psi_{12}: & \Re^{n_1}x\Re^{m_1} & \to & \Re^{n_2+m_2} \\ & (x,x_1) & \to & \left(g_2(x,x_1), h_2(x,x_1)\right) \end{array}$$

and

$$\begin{array}{llll} \Psi_{21}: & \Re^{n_2}x\Re^{m_3} & \to & \Re^{n_1+m_1} \\ & (x',x_2) & \to & \left(g_1(x',x_2), h_1(x',x_2)\right) \end{array}$$

For illustration, let $n_1 = 1$ and $n_2 = 2$. Then complete $\Re$ space and take $m_1 = 1$ and $m_2 = 0$. Define the applications g_1 and g_2 in the following way

$$\begin{cases} g_2: & \Re^2 & \to & \Re^2 \\ & (x,x_1) & \to & g_2(x,x_1) = (x - x_1, x + x_1) \end{cases}$$

and

$$\begin{cases} g_1: & \Re^2 & \to & \Re \\ & x := (x'_1, x'_2) & \to & g_2(x'') = \left(\dfrac{x'_1 + x'_2}{2}\right) \end{cases}$$

Remember that E has a $\{1\}x\Re^{n_1} \cup \{2\}x\Re^{n_2}$ shape and the measure ξ is symmetric on $E \times E$ based on the g$_1$ and g$_2$ applications. Begin by defining measure ξ on then symmetrically on $\Re^{m_1}x\Re^{m_2}$ and finally extending on $E \times E$. Consider a transformation

$$\begin{array}{llll} \varphi: & \Re^{n_1}x\Re^{m_1} & \to & \Re^{n_2}x\Re^{m_2} \\ & (x,x_1) & \to & \left(x, g_2(x,x_1)\right) \end{array}$$

Since ξ is the image of Lebesgue measure λ from $\Re^{n_1}x\Re^{n_2}$ through application then it can be written by $d\xi = \varphi.d\lambda$. For $A \subset \Re^{n_1}$ and $B \subset \Re^{n_2}$ verify

$$\xi(AxB)$$
$$= \lambda\left\{(x,x_1) \in \Re^{n_1}x\Re^{m_1} \middle| x \in A \text{ and } g_2(x,x_1) \in B\right\}$$

This definition is extended to $\Re^{m_1}x\Re^{m_2}$ through symmetrical properties by writing

$$\xi(BxA) = \xi(AxB)$$

for $A \subset \Re^{n_1}$ and $B \subset \Re^{n_2}$. Finally

$$\xi(AxB)$$
$$= \xi\left(A \cap \Re^{n_1}xB \cap \Re^{n_2}\right) + \xi\left(A \cap \Re^{n_2}xB \cap \Re^{n_1}\right)$$

Note that

$$\xi(AxB) = 0 \text{ if } A \subset \Re^{n_1} \text{ and } B \subset \Re^{n_1}$$

and

$$\xi(AxB) = 0 \text{ if } A \subset \Re^{n_2} \text{ and } B \subset \Re^{n_2}$$

For two positive variable functions $\psi(x,y)$ on $E \times E$ verify

$$\iint_{ExE} \psi(x,x')\xi(dx,dx') = \iint_{\Re^{n_1}x\Re^{n_1}} \psi(x,x')\xi(dx,dx')$$
$$+ \iint_{\Re^{n_2}x\Re^{n_2}} \psi(x,x')\xi(dx,dx')$$

Because ξ is symmetric then it also verifies

$$\iint_{ExE}\psi(x,x')\xi(dx,dx')$$

$$= \iint_{\Re^{n_1}x\Re^{n_1}}(\psi(x,x')+\psi(x',x))\xi(dx,dx')$$

Finally, for $A\subset\Re^{n_1}$ and $B\subset\Re^{n_2}$ verify

$$\iint_{AxB}\psi(x,x')\xi(dx,dx')$$

$$= \iint_{ExE}1_A(x)1_B(x')\psi(x,x')\xi(dx,dx')$$

$$= \iint_{\Re^{n_1}x\Re^{n_2}}1_A(x)1_B(g_2(x,x_1))\psi(x,g_2(x,x_1))\xi(dx,dx_1)$$

2.2 Density Function

Let $x\in\Re^{n_1}$. Select a jump to $\Re^{n_2}$ with probability j(2,x) and stay in $\Re^{n_1}$ with probability 1-j(2,x). Take a random point $x_1\in\Re^{m_1}$ with auxiliary distribution q1(x1) and then let $x'=g_2(x,x_1)$.

Let $\pi_1(x)$ and $\pi_2(x)$ be density functions against Lebesgue measures of $\Re^{n_1}$ and $\Re^{n_2}$. Then

$$\int_A \pi(dx)\int_B q(x,dx')\rho(x,x')$$

$$= \iint_{\Re^{n_1}x\Re^{n_2}}1_A(x)\pi_1(x)1_B(g_2(x,x_1)))$$

$$j(2,x)\rho(x,g_2(x,x_1))\ q_1(x_1)dxdx_1$$

with $A\subset\Re^{n_1}$ and $B\subset\Re^{n_2}$. According to the inversion conditions, for x and x' given there is a single x1 such that $x'=g_2(x,x_1)$. Thus $q_1(x_1)$ is denoted by $q_1(x,x')$.

$$\int_A \pi(dx)$$

$$= \iint_{AxB}\pi_1(x)j(2,x)\rho(x,x')\ q_1(x,x')\xi(dx,dx')$$

So the density function of the measure ξ can be written as

$$f(x,x')=\pi_1(x)j(2,x)q_1(x,x')$$

In the same way

$$\int_B \pi(dx')\int_A q(x',dx)\rho(x',x)$$

$$= \iint_{\Re^{n_1}x\Re^{m_1}}1_B(x')\pi_2(x')1_A(g_1(x',x_2))$$

$$j(1,x')\rho(x',g_1(x',x_2))\ q_2(x_2)dxdx_2$$

To write this integral to the measure ξ, make a variable changement

$$\begin{cases} x' &= g_2(x,x_1) \\ x_2 &= h_2(x,x_1) \end{cases}$$

If the integral on the right-hand side is expressed as a function of x and x1 then Jacobian will appear

$$\left|\frac{\partial(x',x_2)}{\partial(x,x_1)}\right|$$

So that

$$\int_B \pi(dx')\int_A q(x',dx)\rho(x',x)$$

$$= \iint_{AxB}1_B(x')\pi_2(x')1_A(x)\ j(1,x')\rho(x',x)$$

$$q_2(x',x)\left|\frac{\partial(x',x_2)}{\partial(x,x_1)}\right|\xi(dx,dx')$$

Then

$$f(x',x)=\pi_2(x')j(1,x')q_2(x',x)\ \left|\frac{\partial(x',x_2)}{\partial(x,x_1)}\right|$$

The probability of acceptance becomes

$$\rho(x,x')=\min\left\{1,\frac{\pi_2(x')j(1,x')q_2(x',x)}{\pi_1(x)j(2,x)q_1(x,x')}\left|\frac{\partial(x',x)}{\partial(x,x_1)}\right|\right\}$$

This reversible jump MCMC algorithm is a method producing an ergodic Markov chain with a stationary distribution. This Markov chain can be considered as a random variable whose distribution is the posterior distribution. Furthermore, this Markov chain is then used to estimate the parameter ψ.

3. RESULTS AND DISCUSSION

The parameter ψ is estimated by using a Bayesian method and a likelihood function is determined.

3.1 Likelihood Function

Because the variable random z_t has an exponential distribution with parameter λ for $t=1,2,\ldots,n$, the density function of z_t is

$$f(z_t|\ \lambda)=\lambda\exp-\lambda z_t \tag{2}$$

The variable transformation

$$x_t=\sum_{i=1}^{p}\phi_i x_{t-i}+z_t \tag{3}$$

is used. Then $z_t=x_t-\sum_{i=1}^{p}\phi_i x_{t-i}$ and $\frac{dz_t}{dx_t}=1$.

Let $x=(x_1,x_2,\ldots,x_n)$ be a realization vector of AR(p) with an exponential error. Thus, the density function of x_t is

$$f(x_t|\ \psi)=\lambda\ \exp-\lambda(x_t-\sum_{i=1}^{p}\phi_i x_{t-i}) \tag{4}$$

for $t=1,2,\ldots,n$.

Suppose that $y_0=(x_1,x_2,\ldots,x_p)$ and $y=(x_{p+1},x_{p+2},\ldots,x_n)$. Then the likelihood function of y can is approximated by:

$$L(y|\ \psi) = \lambda^{n-p}\exp-\lambda\sum_{t=p+1}^{n} g(t,p,\phi^{(p)}) \tag{5}$$

where

$$g(t,p,\phi^{(p)}) = x_t - \sum_{i=1}^{p} \phi_i x_{t-i} \tag{6}$$

for $t = p+1, 2, \ldots, n$, with an initial value y_0. Let S_p be a stationary region and $r^{(p)} = (r_1, r_2, \ldots, r_p)$ be a sample partial autocorrelation vector. By using a transformation

$$F : \phi^{(p)} \in S_p \to r^{(p)} \in [-1,1]^p \tag{7}$$

then the AR(p) model is stationary if and only if $r^{(p)} \in [-1,1]^p$. Finally, the approximated likelihood function of the y can be written by:

$$L(y|\ \theta) = \lambda^{n-p}\exp-\lambda \sum_{i=p+1}^{n} g(t,p,F^{-1}(r^{(p)})) \tag{8}$$

where $\theta = (p, r^{(p)}, \lambda)$ and F^{-1} is an inverse transformation of F.

3.2 Prior Distribution

Before obtaining a posterior distribution, a prior distribution must be selected. The prior distribution is taken as follows. A binomial distribution is chosen for a number of order $p\ (p = 1, 2, \ldots, p_{max})$

$$\pi(p|\varphi) = C_p^{p_{max}}\varphi^p(1-\varphi)^{p_{max}-p} \tag{9}$$

where p_{max} is a maximum of p and μ is an hyper-parameter. A uniform distribution is chosen for a coefficient vector $r^{(p)}$

$$\pi(r^{(p)}|\ p) = U(0,1)^p \tag{10}$$

Also, a uniform distribution is chosen for a parameter λ

$$\pi(\lambda) = U(0,1) \tag{11}$$

Furthermore, a hyper-prior distribution for φ is a uniform distribution.

Let $\pi(\theta,\varphi)$ be a prior distribution for (θ,φ). Because the distribution of θ given φ is $\pi(\theta|\ \varphi) = \dfrac{\pi(\theta,\varphi)}{\pi(\varphi)}$, the prior distribution for (θ,φ) can be written as follows:

$$\pi(\theta,\varphi) = \pi(\theta|\ \varphi)\pi(\varphi) \tag{12}$$

where $\pi(\theta|\ \varphi)$ is a conditional distribution of θ given φ and $\pi(\varphi)$ is a marginal distribution for?

3.3 Posterior Distribution

Let $\pi(\theta,\varphi|\ y)$ be a posteriori distribution for the parameter and the hyper-parameter (θ,φ). According to the Bayesian Theorem, the posterior distribution for (θ,φ) is given as follows

$$\begin{aligned}\pi(\theta,\varphi|\ y) &\propto f(y|\ \theta)\pi(\theta,\varphi)\\ &\propto f(y|\ \theta)\pi(\theta,\varphi)\ \pi(\theta|\ \varphi)\pi(\varphi)\end{aligned} \tag{13}$$

Unfortunately, the Bayesian estimator cannot be determined analytically because the posterior distribution of parameter θ and hyper-parameter φ has a complicated form. To overcome these problems, reversible jump MCMC Algorithm [9] is used.

3.4 Reversible Jump MCMC

Suppose that $M = (\theta,\varphi)$. An MCMC method for the simulation of a distribution $\pi(\theta,\varphi|\ y)$ produces an ergodic Markov chain $M_1, M_2, \ldots, M_m$ whose stationary distribution is $\pi(\theta,\varphi|\ y)$. This Markov chain $M_1, M_2, \ldots, M_m$ can be considered as a random variable whose distribution is $\pi(\theta,\varphi|\ y)$. Furthermore, the Markov chain $M_1, M_2, \ldots, M_m$ is used to estimate the parameter M. To realize this, the Gibbs sampling algorithm is adopted. It consists of three steps:

- Simulate $\varphi \sim B(p+1, p_{max}-p+1)$
- Simulate $\lambda \sim G(\alpha,\beta)$ with $\alpha = n-p+1$ and
$$\beta = \sum_{t=p+1}^{n}(x_t - \sum_{i=1}^{p} F^{-1}(r_i)x_{t-i})$$
- Simulate $(p, r^{(p)}) \sim \pi(p, r^{(p)}|\ y, \lambda, \varphi)$

Unfortunately, the distribution $\pi(p, r^{(p)}|\ y, \lambda, \varphi)$ is not an explicit form. The exact simulation cannot possibly be done. Since the value p is not known, the MCMC algorithm cannot be used to simulate $\pi(p, r^{(p)}|\ y, \lambda, \varphi)$. Hence the reversible jump MCMC algorithm [9] is adopted.

Let $\omega = (p, r^{(p)})$ be an actual point of the Markov chain. There are 3 types of transformations used, namely: a birth of the order; a death of the order; and a change of the order. Furthermore, let N_p be the probability of the transformation from p to p + 1, let D_p be the probability of transformation from p + 1 to p, and let C_p be the probability of transformation from p to p.

3.4.1 Birth/Death of the Order

A transformation of the birth of the order will change a number of coefficients, from p to the p + 1. Suppose that $\omega = (p, r^{(p)})$ is a current point and $\omega^* = (p+1, r^{(p^*)})$ is an updated point. The birth of the order from $\omega = (p, \phi^{(p)})$ to $\omega^* = (p+1, r^{(p^*)})$ is defined in the following way. Set $p^* = p+1$ and choose a random point $\nu \sim U(-1,1)$. Next, create a new point $\omega^* = (p+1, r^{(p^*)})$ with

$$r_1^* = r_1 \text{ , ..., } r_{p^*-1}^* = r_p \text{ , } r_{p^*}^* = \nu$$

Otherwise, the transformation of the death of the order will change the number of coefficients, from p+1 to p. Suppose that $\omega = (p+1, r^{(p+1)})$ is a current point and $\omega^* = (p+1, r^{(p^*)})$ is an updated point. The death of the order from $\omega = (p+1, \phi^{(p+1)})$ to $\omega^* = (p, r^{(p^*)})$ is defined in the following way. Set $p^* = p$ and create a new point $\omega^* = (p, r^{(p^*)})$ with

$$r_1^* = r_1 \text{ , ..., } r_{p^*}^* = r_p$$

Suppose that a_n and a_d are respectively a probability of acceptance for the birth of the order and death of the order. The probability of acceptance for birth is as follows:

$$a_n(\omega, \omega^*) = \min\left\{1, \frac{\pi(\omega^* \mid \varphi, y)}{\pi(\omega \mid \varphi, y)} \frac{q(\omega_*, \omega)}{q(\omega, \omega^*)}\right\}$$

While the probability of death is as follows:

$$a_d(\omega, \omega^*) = \min\left\{1, \frac{1}{a_n(\omega^*, \omega)}\right\}$$

where

$$\frac{\pi(\omega^* \mid \varphi, y)}{\pi(\omega \mid \varphi, y)} = \frac{D_{k+1}}{N_k} \frac{(\beta^*)^{n-p}}{\beta^{n-p+1}} \frac{1}{n-p} \frac{p+1}{p_{max} - p}$$

and

$$\frac{q(\omega^*, \omega)}{q(\omega, \omega^*)} = 1.$$

3.4.2 Change of the Coefficients

The transformation of the change of order will not change the number of coefficients. This transformation will change the value of coefficients. Suppose that $\omega = (p, r^{(p)})$ is a current point and $\omega^* = (p, r^{(p^*)})$ is an updated point. The change of the coefficients from $\omega = (p, \phi^{(p+1)})$ to $\omega^* = (p, r^{(p^*)})$ is defined in the following way. Set $p^* = p$, choose a random point and choose a random point $u \sim U(-1,1)$. Then a new point $\omega^* = (p, r^{(p^*)})$ is created with $r_1^* = r_1$,..,

$$r_{i-1}^* = r_{i-1}, \; r_i^* = u, \; r_{i+1}^* = r_{i+1}, \; r_{p^*}^* = r_p$$

Let a_p be a probability of acceptance of a change of coefficient. Then the probability of acceptance of change is as follows:

$$a_p(\omega, \omega^*) = \min\left\{1, \frac{\pi(\omega^* \mid \varphi, y)}{\pi(\omega \mid \varphi, y)} \frac{q(\omega_*, \omega)}{q(\omega, \omega^*)}\right\}$$

where

$$\frac{\pi(\omega^* \mid \varphi, y)}{\pi(\omega \mid \varphi, y)} = \left(\frac{\beta^*}{\beta}\right)^{\alpha}$$

and

$$\frac{q(\omega_*, \omega)}{q(\omega, \omega^*)} = 1.$$

The transformation in Eq. (7) is used in order to get the stationary conditions for an AR model. Thus the first result of this paper is an AR model that is obtained that is always stationary. The hierarchical Bayesian was adopted to estimate the order of the AR model, the coefficient of the AR model, the variance of the white noise, and its hyper-parameter. The second result of this paper is that both the order of the AR model and the coefficient of the AR model, the variance of the white noise, and the hyper-parameter can be estimated simultaneously.

4. CONCLUSION

The purpose of this paper was to estimate the parameters of an AR model with exponential white noise when the order was unknown. The parameters cannot be estimated by a Markov chain Monte Carlo algorithm, because the order of the AR model is unknown.

The reversible jump Markov chain Monte Carlo algorithm is one of the new methods that can be used to estimate the parameters of AR models when the order of the AR is unknown. The advantage of this method is that both the order of the AR and the coefficient of the AR can be estimated simultaneously.

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