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NEARLY-OPTIMUM SELECTION OF STRENGTH t ORTHOGONAL ARRAYS: A STRENGTH AND EFFICIENCY APPRAISAL

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ABSTRACT: The process of enumeration and construction of orthogonal arrays has been extensively studied and several methods to develop and implement them have been proposed. There is, however, a gap in the literature when deciding the most appropriate orthogonal array to be tailored to a specific situation. Romero and Murray [1] presented a combinatorial-based technique to list all the possible isomorphic arrays for a given design type and undertook the enumeration of almost all of them (up to 100 runs). The size of the orbits under consideration (the number of isomorphic arrays or possible arrangements for the same design type) was found very large. Certainly, the decision to select the most suitable array for a given situation becomes a non-trivial and computationally intensive task. Some attempts to overcome this problem have been proposed [2] [3], yet a context-based approach has not fully been taken into consideration. In this paper, an engineering-based approach is presented to select orthogonal arrays according to their isomorphism and experimental implementation. The arrays were simulated and tested using a protective relay and its associated circuits. Although still in the process of testing, preliminary results show important performance advantages over traditional techniques such as crossed arrays, combined arrays and response surface methods.

Keywords: Experimental Design, Orthogonal Arrays, Engineering Design, Combinatorics.

1. INTRODUCTION

The present study deals with combinatorial designs (orthogonal arrays) and their implementation in engineering parameter design. An orthogonal array is a multiset whose elements are the different combinations of factor levels (discrete values of the variable under study), having well-defined orthogonality properties [4]. The applied methodology blends the fields of engineering, statistics, combinatorics, group theory, and backtrack search. The focus is on the selection of nearly-optimal orthogonal arrays according to an engineering-based approach rather than a pure mathematical framework. The theoretical background based on linear algebra is presented in section 2. Section 3 introduces the fundamentals of orthogonality from a pure mathematics point of view. Section 4 presents the selection of the arrays. A case study is shown in section 5 alongside statistical considerations and analysis presented in sections 6 and 7 respectively.

2. THEORETICAL BACKGROUND

Define $\boldsymbol{F_{ND}}$ to be the set of all fractional factorial designs (FFDs) with design type $U=(N,s^d)$. Here, N is the number of treatments $S=\{0,\cdots,s-1\}$ and $\boldsymbol{F_{ND}} =\{F:F$ is a $N\times d$ matrix, with each $F_{ij}\in S\}$. Define F in $\boldsymbol{F_{ND}}$ of strength t and frequency γ if any $N\times t$ sub-matrix of F contains all possible row vectors with the same frequency [**4**]. A FFD with strength $t\geq 1$ is called an *orthogonal array*. Now define a mixed design type $U=(N,s_1^{a_1}\cdots s_m^{a_m})$ with $d=\sum_{k=1}^{m}a_k$; and define $S_k=\{0,\cdots,s_k-1\}$ so that $s_k=|S_k|$. Also, define the *set of classes* as $J_k=\{a_1+\cdots+a_{k-1}+1,\cdots,a_1+a_{k-1}+a_k\}$. Similarly, define $R_j=S_k$; where k is the unique index with $j\in J_k$.

Let $\boldsymbol{F_{ND}}$ be the set of all fractional factorial designs with design type U, hence $=\{F:F$ is a $N\times d$ matrix, with $F_{ij}\in R_j \quad \forall i,j\}$. Define now a *Mixed Orthogonal Array* (OA) to be an F in $\boldsymbol{F_{ND}}$ with elements from $s_1,s_2,\cdots,s_k$ with two elements or more, size *N*, *d* factors, and $s_k=|S|$ levels. This array is described as an orthogonal array of the *design type* $U=(N,s_1^{a_1}\cdots s_m^{a_m})$.

Consider two FFDs to be *isomorphic* if one can be obtained from the other by performing a sequence of column, row and symbol permutations. Denote the *set of experimental units* by $\Omega=\{y_1,y_2,\cdots,y_k\}$, and denote the runs as $\{\rho_1,\cdots,\rho_N\}$. Also, define a *treatment* to be the individual combinations of factor levels applied to an experimental unit in each run. The set of all possible treatments is identified as τ; whereas its elements (actual *treatments*) are described with lower-case Latin letters such as *i*.

Let Ω be a set of experimental units (EUs) and τ a set of possible treatments for these units. Let's define a *design* as a map ϕ from Ω to τ, ie $\phi:\Omega \to \tau$. Each experimental unit $y_i \in \Omega$ is assigned to a treatment $\tau_i \in \tau$ through ϕ. The map $\phi:\Omega \to \tau$ is called a *treatment mapping*. Thus, $\tau_i = \phi(y_i)$.

Assume two factors γ_1 and γ_2 with a number of levels s_{γ_1} and s_{γ_2} respectively. Let W_{γ_1} be a subspace of $V_{\gamma_1}^S$ and W_{γ_2} a subspace of $V_{\gamma_2}^S$. The subspace W_{γ_1} becomes $W_{\gamma_1} = V_{\gamma_1}^S \cap V_0^{\perp}$, and the subspace W_{γ_2} becomes $W_{\gamma_2} = V_{\gamma_2}^S \cap V_0^{\perp}$, where V_0 is the subspace of V^S with a basis $\{\mathbf{u_0}\}$ and $\dim V_0 = 1$. Moreover, it follows that $\dim V_{\gamma_1}^S = |s_{\gamma_1}|$, $\dim W_{\gamma_1} = |s_{\gamma_1 - 1}|$, $\dim V_{\gamma_2}^S = |s_{\gamma_2}|$, and $\dim W_{\gamma_2} = |s_{\gamma_2 - 1}|$.

Let $\{\mathbf{u}_1 \ldots \mathbf{u}_N\}$ be an orthogonal basis for V^S; thus, for each treatment τ_i in τ, $\mathbf{u}_i \cdot \mathbf{u}_i = r_i$, with $\sum r_i = N$. Moreover, if $i \neq j$ then $\mathbf{u}_i \perp \mathbf{u}_j$ and the set of vectors $\{\mathbf{u}_i : i \in \tau\}$ is an *orthogonal basis* for W.

Let γ_1 and γ_2 be factors, and let W_{γ_1} and W_{γ_2} be the corresponding subspaces. Thus, if every combination of factor levels γ_1 and γ_2 occurs on the same number of runs, then, In addition, let γ_1 and γ_2 be factors and $V_{\gamma_1}^S, V_{\gamma_2}^S$, $W_{\gamma_1}, W_{\gamma_2}$ the corresponding vector spaces and subspaces. Thus, $V_{\gamma_1} + V_{\gamma_2} = V_o \oplus W_{\gamma_1} \oplus W_{\gamma_2}$ and $\dim(V_{\gamma_1}^S + V_{\gamma_2}^S) = 1 + (s_{\gamma_1} - 1) + (s_{\gamma_2} - 1)$.

2.1 Analysis of Variance

Suppose that the comparison of N different treatments for the vector of treatments τ needs to be done. This *experiment* would be represented by a mathematical *model* as the one shown in equation (1):

$$y_i = \mu + \tau_{i,s} + (\tau_{i,s}\,\tau_{j,s}) + \varepsilon_i, \tag{1}$$

where $i \in \{1,2,\ldots,N\}$ and $j \in \{1,2,\ldots,d\}$; μ is the *overall mean effect,* and the term ε_i is the *random error* component made up of all sources of unexplained variability such as uncontrolled factors and differences between EUs. $\tau_{i,s}$ represents the main effect of factors γ_i with S levels, and $\tau_{i,s}\,\tau_{j,s}$ is the term for the interaction among the factors.

Let R and W be subspaces of V^S, and let $\mathbf{Y}$ be the vector of output combinations. Define the *sum of squares* for R as the norm (squared length) of the orthogonal projection of $\mathbf{Y}$ into R written as $\|\mathbf{P}_R\mathbf{Y}\|^2$. Similarly, the sum of squares for W is given by $\|\mathbf{P}_W\mathbf{Y}\|^2$.

From the previous definitions, if follows that the sum of the orthogonal projections for R and W equals to:

$$\|\mathbf{Y}\|^2 = \|\mathbf{P}_W\mathbf{Y}\|^2 + \|\mathbf{P}_R\mathbf{Y}\|^2. \tag{2}$$

Thus, equation (2) is the *total sum of squares* and measures the *overall* variability in the data. Moreover, the expressions $\|\mathbf{P}_W\mathbf{Y}\|^2$ and $\|\mathbf{P}_R\mathbf{Y}\|^2$ are the sum of squares of treatments and residuals respectively.

The *mean square* for treatments and error are calculated through the quotient between their sum of squares and respective dimensions:

$$= \|\mathbf{P}_W\mathbf{Y}\|^2 / \dim W. \tag{3}$$

$$= \|\mathbf{P}_R\mathbf{Y}\|^2 / \dim R. \tag{4}$$

A close examination of the *expected values* of the mean squares given in equations (3) and (4), leads to express the *expected mean square* as

$$MS_{Errors} = \|\mathbf{P}_R\mathbf{Y}\|^2 / N(d-1) = \sigma^2. \tag{5}$$

Following the previous discussion, the equation (1) is rewritten according to a *vector* of observations $\mathbf{Y}$, the corresponding *expectation* $E(\mathbf{Y})$ and the *errors* ε, having $E(\varepsilon)=0$ and $var(\mathbf{e})=\sigma^2$. Thus,

$$\mathbf{Y}=E(\mathbf{Y})+\varepsilon \tag{6}$$

$$=\mu+EC(\mathbf{Y});\quad \text{var}(\mathbf{Y}=\sigma_Y^2), \tag{7}$$

where $EC(\mathbf{Y})$ is the *mean deviation* from μ within the corresponding class J_k.

The overall mean μ and all of the components of EC $(\mathbf{y})$ are estimated using the *Generalised Least Squares* (GLS) method. It is assumed, for all of the models, that all the random variables (called from now on *design parameters*) are mutually independent and have expectation zero. In addition, it is also assumed that the variance σ^2 of the error terms is the same in all of the different factor combinations [7].

3. ABOUT ORTHOGONALITY

Consider a linear model with two explanatory variables X_1 and X_2, N observations (subjects or experimental units), where $j=1,\ldots,N$. Thus [8],

$$y_j=\beta_0+\beta_1\hat{X}_{1j}+\beta_2\hat{X}_{2j}+\varepsilon, \tag{8}$$

where $\hat{X}_{1j}=X_{1j}-\overline{X}_i$.

The variances and the covariance are obtained as follows:

$$V(b_0)=\frac{\sigma^2}{N}, \tag{9}$$

$$V(b_1)=\left\{\frac{1}{(1-\cos^2\theta)S_1^2}\right\}\frac{\sigma^2}{N}, \tag{10}$$

$$V(b_2)=\left\{\frac{1}{(1-\cos^2\theta)S_2^2}\right\}\frac{\sigma^2}{N},$$

$$(b_0,b_1)=0,\qquad (b_o,b_2)=0,$$

$$(b_1,b_2)=\left\{\frac{-\cos\theta}{(1-\cos^2\theta)S_1S_2}\right\}\frac{\sigma^2}{N}; \tag{11}$$

where

$$S_1^2=N^{-1}\sum\hat{X}_{1j}^2,\qquad S_2^2=N^{-1}\sum\hat{X}_{2j}^2,$$

$$\cos\theta=\frac{\sum\hat{X}_{1j}\hat{X}_{2j}}{\left\{\left(\sum\hat{X}_{1j}^2\right)\left(\sum\hat{X}_{2j}^2\right)\right\}^{1/2}}, \tag{12}$$

and all of the sums are over $j=1,\ldots,N$.

Note that S_1 and S_2 are measures of the spread against the desired design point towards the X_1 and X_2 directions; while θ is the angle between the design vectors $\hat{X}_1=(\hat{X}_{11},\hat{X}_{12},\ldots,\hat{X}_{1N})^T$ and $\hat{X}_2=(\hat{X}_{21},\hat{X}_{22},\ldots,\hat{X}_{2N})^T$. The aim here is to reduce the variances given in Equation (10) as much as possible. This is done by either making S_1^2 and S_2^2 large values or making $\cos^2\theta$ as small as possible. At the extreme, when $\cos\theta=0$ the *n-dimensional* design vectors $\hat{X}_1$ and $\hat{X}_2$ are at *right* angles; that is, they are *orthogonal* to each other. The aforementioned condition can be met by making the sum $\sum\hat{X}_{1j}\hat{X}_{2j}=0$. Moreover because $\cos\theta$ is an actual measure of the correlation between the two vectors X_1 and X_2, implies that they are *uncorrelated*. That is, an orthogonal array satisfies the condition that *factors represented through columns are uncorrelated.* This is one of the main features in *parametric design* discussed in this paper.

4. THE SELECTION OF THE ARRAYS

When dealing with two isomorphic designs, say $F_1\cong F_2$, it is expected changes in the order of the runs (and eventually, how the actual experiment will be conducted). Similarly, using two non-isomorphic designs (orbit representatives) it is expected some changes of the actual treatments τ_i to be implemented. When the calculations of the sum of squares are performed (main effect and residual), it is not possible to discriminate subtle changes in the order of the runs. However, the detection of changes in the treatments through the corresponding sum of squares can be done.

When dealing with fractional factorial designs, the equation (7) is used in its matrix equivalent as follows:

$$\mathbf{y}=\beta\cdot\mathbf{X}+\varepsilon. \tag{13}$$

$\mathbf{y}$, the outcome and the vector of errors ε have both dimensions N. Furthermore, an array X has

$N \times k$ elements, whereas the vector of *coefficients* β has $k \leq N$ elements. The estimators of the coefficients β are calculated using the *least square* technique as:

$$\hat{\beta} = (X^T X)^{-1} X^T \mathbf{y}. \quad (14)$$

Note the importance of the term $(X^T X)^{-1} X^T$ in equation (14), as by minimizing this term allows creating *optimal factorial designs*.

For a full capacity estimation of the experiments, the design should permit the estimation of all main effects and some interactions. Feasible design options for the orthogonal arrays are the *maximum strength compatible with run-size and factor specifications*, and choosing an array which *maximizes the D-efficiency for the variables and replicates of the full model*. That is, the selection of the arrays will depend upon two criteria; namely, *strength is the best* and *D-efficiency is the best*. For the first criterion, all representative arrays (non-isomorphic) having a specific design type *U* are generated. From this initial selection, the arrays with full estimation capacity of the main effects are chosen. Finally, the selection of the array with maximum D-efficiency is done. Regarding the former criterion, some arrays are randomly generated (according to the corresponding orbit size) for further improving the D-efficiency by using the modified Fedorov algorithm [11]. Then, the one that performs the best is chosen.

5. THE CASE STUDY

Successful operation of a power system, an industrial operation, or an isolated electric motor, mainly depends on the maintenance of an adequate insulation for all living and conducting parts [9]. For an electric system to continue to deliver its desired performance against a potential fault, it its very important that the section on fault be isolated promptly from the rest of the system. It is the task of the protective relays and their associated circuit breakers to accomplish this.

In this case study, a statistical approach to engineering design is used by dealing with a simplified version of a protective relay (see Figure 1 adapted from [9] and [10]).

The device under consideration is an electromechanical switch which operates when the winding circuit is closed. When doing relay design, two main types of problems arose: the relay's mechanical construction and its electrical performance. Consider in this study that the design parameters (DPs) are random variables which are classified as follows:

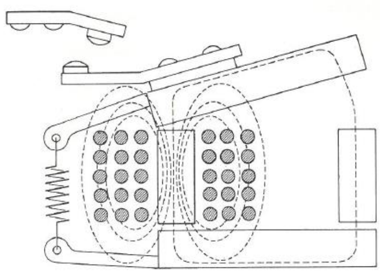

Fig. 1 Hingeless Relay.

• DPs associated with the mechanical circuit (for instance, armature and spring.)
• DPs associated with the magnetic circuit (for instance, coil, permeability features, magnetic reluctance, and ampere-turns.)
• DPs associated with the electrical circuit (coil resistance, supplied voltage, and current.)

The main functional requirement is a *satisfactory contact performance*. This functional requirement is physically represented as the resistance vs force relationship of contact alloys and their variations with surface conditions. Thus, the resultant force requirements relay on the mechanics of the *contact springs* associated with a pulling force.

6. STATISTICAL CONSIDERATIONS

6.1 Mechanical circuit

This circuit includes the contacts, armature to operate the magnetic field, and the spring. Note that mechanical parameters defined by the physical geometry of the device are random variables (beam length, cross-sectional area, a moment of inertia, and contact area). This circuit accounts for the total load variation $(\bar{p}, S_p)$.Using the second Newton's Law, the total load variation equals the *contact force variate* $(\bar{p}_1, S_{p_1})$ plus the deflection force variate $(\bar{p}_2, S_{p_2})$.

6.2 Electrical circuit

The random variables for this system are voltage source (because of voltage variations) and coil resistance (due conductor resistivity and geometry, among other factors.) Define the *voltage variable* as E and the resistance R. The model considers the power delivered by a battery of 30V. The voltage standard deviation S_E, is estimated as 2% of the nominal voltage; therefore $(\bar{E}, S_E) = (30, 0.6)[\mathrm{V}]$. In the following discussion, consider the random variable *coil resistance* to be determined by the random variables resistance and length of the wire. The *wire length* is a random variable made upon the wire diameter, its insulation thickness variability, and the spool geometry.

6.3 Magnetic circuit

The magnetic field strength is a function of coil features, magnetic characteristics of the core material, the return circuits, and gap features. Thus, the relations among associated random variables are expressed in terms of the vector for flux density (induction) B and the vector for the field intensity H.

The design parameters are as follows:

- L_1 : center line length of the return path, [cm].
- L_2 : center line length of the coil core, [cm].
- a : cross-sectional area of the return path and coil core, $[\text{cm}]^2$.
- x : main gap separation when the armature is closed, [cm].
- A : effective pole face area multiplied by μ_0 (permeability of the air), $[\text{cm}]^2$.

The *pulling force* is given by [139]:

$$F = \frac{2\pi \cdot NI^2}{A(R_0 + \frac{x}{A})^2}. \tag{15}$$

7. ANALYSIS

The orthogonal arrays were implemented according to magnetic, mechanical, and electric design parameters of the relay. These results are summarised as follows:

7.1 Mechanical Circuit

Random Variables: contact force, deflection force.

• Contact force:
$(\bar{\boldsymbol{p}}_1, \boldsymbol{S}_{p_1}) = (\mathbf{2.60}, \mathbf{0.20})$ [g]
• Deflection force:
$(\bar{\boldsymbol{p}}_2, \boldsymbol{S}_{p_2}) = (\mathbf{53.14}, \mathbf{6.75})$ [g]

7.2 Electrical Circuit

Random Variables: a voltage source, coil resistance, and wire length (measured by its overall diameter OD).

• Voltage:
$(\bar{\boldsymbol{E}}, \boldsymbol{S}_E) = (\mathbf{30}, \mathbf{0.6})[\mathbf{V}]$
• Coil resistance:
$(\bar{\boldsymbol{\rho}}, \boldsymbol{S}_\rho) = (\mathbf{0.0905}, \mathbf{0.00226})[\boldsymbol{\Omega}/\mathbf{in}]$
• Overall diameter:
$(\overline{\boldsymbol{OD}}, \boldsymbol{S}_{OD}) = (\mathbf{0.035}, \mathbf{0.0001})[\mathbf{in}]$
• Coil geometry:
$(\bar{\boldsymbol{t}}_L, \boldsymbol{S}_{t_L}) = (\mathbf{84.86}, \mathbf{2.51})$
• No of layers of turns:
$(\bar{\boldsymbol{L}}_N, \boldsymbol{S}_{L_N}) = (\mathbf{37.71}, \mathbf{1.12})$
• Wire length:
$(\bar{\boldsymbol{L}}_t, \boldsymbol{S}_{L_t}) = (\mathbf{4.384}, \mathbf{57.60})[\mathbf{in}]$
• Resistance:
$(\bar{\boldsymbol{R}}, \boldsymbol{S}_R) = (\mathbf{396.7}, \mathbf{7.1117})[\boldsymbol{\Omega}]$
• Coil current:
$(\bar{\boldsymbol{I}}, \boldsymbol{S}_I) = (\mathbf{0.0756}, \mathbf{0.0026})[\mathbf{A}]$

7.3 Magnetic Circuit

Random Variables: Gap with the relay coil energized, cross-sectional area, the center of the line length of the return path, center line length of the coil core, effective pole face area, permeability, un-operated gap.

• Gap*:*
$(\bar{\boldsymbol{X}}_G, \boldsymbol{S}_{X_G}) = (\mathbf{0.0457}, \mathbf{0.00042})[\boldsymbol{cm}]$
• Cross-sectional area*:*
$(\bar{\boldsymbol{a}}, \boldsymbol{S}_a) = (\mathbf{0.019}, \mathbf{0.0053})[\boldsymbol{cm}]^{\mathbf{2}}$
• Centre of the line length of the return path*:*
$(\bar{\boldsymbol{L}}_1, \boldsymbol{S}_{L_1}) = (\mathbf{5.387}, \mathbf{0.1223})[\boldsymbol{cm}]$
• Centre line length of the coil core*:*
$(\bar{\boldsymbol{L}}_2, \boldsymbol{S}_{L_2}) = (\mathbf{0.754}, \mathbf{0.008})[\boldsymbol{cm}]$
• Effective pole face area*:*
$(\bar{\boldsymbol{A}}, \boldsymbol{S}_A) = (\mathbf{0.016}, \mathbf{0.001})[\boldsymbol{cm}]^{\mathbf{2}}$
• Permeability:
$(\boldsymbol{R}_0, \boldsymbol{S}_0) = (\mathbf{2.894}, \mathbf{0.3162})$
• Un-operated gap:
$(\bar{\boldsymbol{X}}, \boldsymbol{S}_X) = (\mathbf{0.113}, \mathbf{0.013})[\mathbf{cm}]$
• Random variable to calculate the pulling force:
$(\boldsymbol{R}_0 + \boldsymbol{X}/\boldsymbol{A}) = (\mathbf{9.950}, \mathbf{0.328})$

7.4 Design to a specified reliability

This case study deals with a level of reliability required for the relay of $R > 0.999999$ calculated using the method shown in this paper. By keeping *all of the previous design parameters unchanged* (except for the level of voltage E), the specified level of R can be worked out. This is done starting from the given random variable *load*: $(\bar{\boldsymbol{\rho}}, \boldsymbol{S}_\rho) = (55.74, 6.76)[\text{g}]$.

Note that the force needed to depress the spring and exert the required contact pressure is kept unchanged. Thus, the *pulling force* in terms of the voltage E is now written as:

$$F = \frac{2\pi N^2}{A(R_0 + x/A)^2}\left(\frac{E}{R}\right)^2 \frac{1}{980}.$$

The product variate estimate of $\boldsymbol{R^2}$ and $\boldsymbol{A(R_0 + x/A)}$ is calculated as $\left(\mathbf{3.823 \cdot 10^5, 1.002 \cdot 10^4}\right)$. The pulling force is expressed as a function of the voltage $\boldsymbol{E}$ as follows:

$$(\bar{F}, S_F) = (0.205\bar{E}^2, 0.0877\bar{E}^2)[\mathrm{g}]$$

The previous result comes from the fact that

$$S_E^2 = 3\bar{E}S_E \text{and } S_E \approx 0.1\bar{E}.$$

Recalling that

$$\left(\bar{\rho}, S_\rho\right) = (55.74, 6.76),$$

substitute for $\boldsymbol{z} \approx \mathbf{1.5}$. Thus,

$$\bar{E} = 27.7[V].$$

This is the combinatorial-statistical value of the voltage needed to achieve a reliability level of $\boldsymbol{R} > \mathbf{0.999999}$ according to the previous estimated pulling-force parameters of

$$(\bar{F}, S_F) = (248.3, 3.3254)[\mathrm{g}]$$

8. CONCLUSION

A context-based algebraic technique to produce a nearly-optimal orthogonal array was mathematically developed and applied. The selection was based on the strength and D-efficiency of the arrays, alongside the consideration of the corresponding orbits and suitable run sizes to be able to meet the independence criterion. The independence of the design parameters and associated system requirements was achieved by considering the relay's actual operation using the pulling force. Similarly, its efficiency was appraised according to the level of reliability needed in this type of electrical systems.

Preliminary results show that the optimized array has a lower signal to noise ratio than the traditional designs. However, the construction of the arrays, their classification in an orbit, and the selection of the best design do require more computational power and considerable more time. The authors are currently working on fixing these two main issues.

9. ACKNOWLEDGEMENTS

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