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HARMONIC WAVES ON AN ABRUPT TRANSITION

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ABSTRACT: A simple long-wave reflection and transmission over an abrupt depth change with constant channel width are presented. Firstly, the wave propagation is modeled on a two-step abrupt transition, and the waves are reflected and transmitted only once. The model is extended to include more than one re-reflection and retransmission as well as depth-limited breaking-wave height criteria. The Dean beach profile is also modeled. The profile is a function of the median grain size of the beach material. It is found that the wave energy is conserved when the waves are re-reflected and retransmitted more than five times. The breaking waves reduce the reflection coefficient by 30%. The results are compared with other research on the reflection coefficient occurring in a smooth sloping beach model. On a small sloping beach, an abrupt depth change gives a significant difference in the value of the reflection coefficient. The reflection coefficient on the smooth small sloping beach is close to zero, while the abrupt depth change can increase the reflection coefficient to about 60% in this case.

Keywords: Harmonic wave, Abrupt transition, Equilibrium beach profile, Reflection Coefficient.

1. INTRODUCTION

A simple long-wave reflection and transmission model is presented. The model is intended to calculate the reflection coefficient resulting from interactions between incident waves and reflected waves on a sloping bottom with and without a breaking wave. The model follows the definitions of reflected and transmitted waves as presented by Dean and Dalrymple (1984) with an abrupt depth change or a step with constant channel width. This model is then extended to include multiple steps and depth-limited breaking waves [1].

Macaskill (1977) presented a method for calculating reflected waves of plane waves propagating over different constant depths, where the intermediate region is considered linear in the cross-shore direction [2]. He considered the fluid motion as complex velocity potential with the kinematic boundary conditions of normal velocity and free surface conditions. The reflection coefficient of maximum 0.5 was found for a depth ratio of 0.5 for one-step abrupt transition case.

Dhillon et al. (2016) investigated a stepped-type bottom topography assuming linear theory [3]. The relationships of the reflection and transmission coefficients with the depth parameter and dock lengths were investigated. He found that a 1 m increase in dock length produced almost 25% more reflected waves. Watson (2012) conducted 40-day field measurements on a steep beach with two different slopes with no underwater bars located at Carmel Beach, California [4]. The nearshore slope was 1/7.6 and the outer slope was 1/19. A highly reflective beach was observed regardless of tidal effect. The beach experienced long-period, low-amplitude sea-swell waves coming from west to northwest. The field measurement results showed the highest reflection coefficient during the long period and low waves are as much as 80%. This high reflected coefficient was also found during high tide.

Shibayama (2009) gives lists of existing breaker-height formulas [5]. The mechanism of wave breaking during propagation toward the nearshore can adequately explain the reflected and transmitted waves over abrupt transition studied.

Walton (1991) studied wave reflection from natural beaches [6]. He conducted field measurements of random waves reaching the nearshore. The reflection coefficient was processed from the wave data records in terms of wave frequencies. He found that the reflection coefficients are produced by the small frequencies. Reflection coefficients produced by waves with frequencies between 0.01 and 0.05 Hz are 0.75–0.95. Waves at bigger wave frequencies have a reflection coefficient of around 0.25 on average.

This paper focuses on propagated waves that are reflected and transmitted only once; this model is called the progressive model. The model is also extended to include waves over a multi-step abrupt transition. Then the reflected and transmitted waves are retransmitted and re-reflected; this model is called the re-reflected wave model.

Finally, the progressive wave model and re-reflected wave model are combined to produce a complete model in which the total energy of the reflected and transmitted waves depends on how many times the waves are transformed by being either reflected or retransmitted.

2. PROGRESSIVE MODEL

This section focuses on waves that are reflected and transmitted once. Figure 1 shows the geometry of the depth transition region. The fluid domain is divided into Regions 1 and 2, as shown. The incoming wave H_i will be assumed to propagate in the positive x-direction. At the vertical step, located at $x = x_1$, a portion of the wave will be reflected, and the remainder is transmitted.

By assuming linear superposition, the wave in Fig.1 is described as follows:

$$\eta_1 = \eta_i + \eta_{r1} = \frac{H_i}{2}\cos\left(k_1 x - \omega\, t + \varepsilon_i\right) + \frac{H_{r1}}{2}\cos\left(k_1 x + \omega\, t + \varepsilon_{r1}\right) \quad \text{at} \quad x < x_1 \qquad (1)$$

$$\eta_2 = \eta_{t1} = \frac{H_{t1}}{2}\cos\left(k_2 x - \omega\, t + \varepsilon_{t1}\right),\ x \ge x_1$$

where η_i, η_{t1}, and η_{r1} are the incident, transmitted, and reflected waves, respectively. ε_i, ε_{t1}, and ε_{r1} are the corresponding wave phases. The phases are referenced to the incident wave, whose phase is set to zero. k_1 and k_2 are the wave numbers before and after the step at $x = x_1$. The wave numbers are calculated using the long-wave dispersion condition as follows:

$$k_i = \frac{2\pi}{\sqrt{gh_i}\ T_p} \qquad (2)$$

where are the wave period and h is the water depth? There are four unknowns: H_{t1}, H_{r1}, ε_{t1}, and ε_{r1}. The matching boundary conditions at the location of the step as shown in Fig.1 could be imposed. The first condition is that the free surface is continuous at $x = x_1$. The condition is given in Eq. (3):

$$\eta_i + \eta_{r1} = \eta_{t1} \qquad ; \quad x = x_1 \qquad (3)$$

For the second condition, the linearized continuity equation is used:

$$\frac{\partial \eta}{\partial t} = \frac{\partial (uh)}{\partial x} \qquad (4)$$

where u is the depth-averaged horizontal velocity. From (3) and (4), it follows that the volume flux must match at the step.

$$(uh)_1 = (uh)_2 \qquad (5)$$

For a long wave, the depth-averaged velocity can be written as [1]

$$u = \frac{\eta c}{h} \qquad (6)$$

where c is the wave celerity. In the direction of the wave, Eq. (5) can be rewritten to Eq. (7):

$$c_1\left(\eta_i - \eta_{r1}\right) = c_2\ \eta_{t1} \qquad (7)$$

For the shallow water, the celerities are

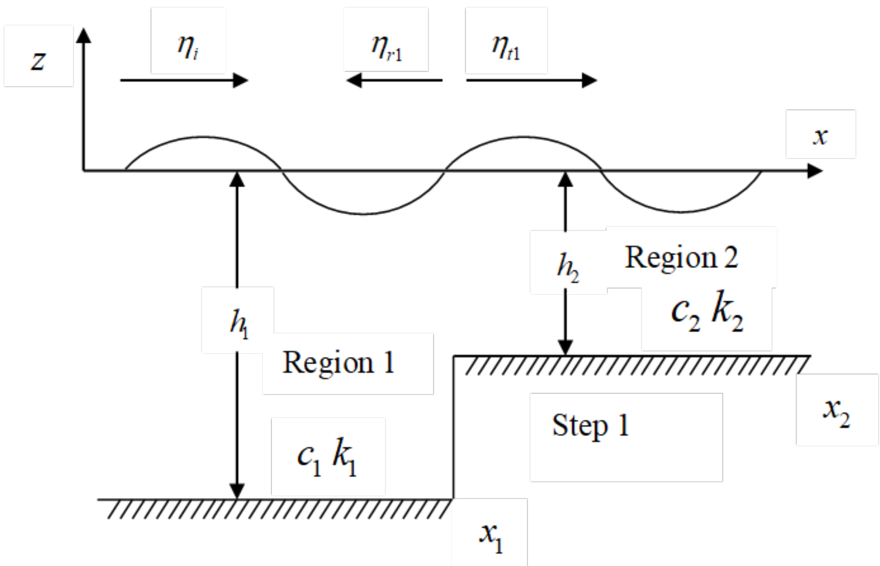

Fig.1 Elevation of a section of one-step abrupt transition.

$$c_1 = \sqrt{g\ h_1}$$
$$c_2 = \sqrt{g\ h_2} \tag{8}$$

Equations (3) and (7) are used to solve the unknowns H_{t1}, H_{r1}, ε_{t1}, and ε_{r1}. The wave heights and phases are written as:

$$\begin{aligned} H_{t1} &= 2\frac{c_1}{c_1+c_2}H_i \\ H_{r1} &= \frac{c_1-c_2}{c_1+c_2}H_i \\ \varepsilon_{t1} &= k_1x_1 - k_2x_2 \\ \varepsilon_{r1} &= -2k_1x_1 \end{aligned} \tag{9}$$

The incident, transmitted, and reflected waves at first step can be written as:

$$\begin{aligned} \eta_i &= \frac{H_i}{2}\cos\left(k_1x-\omega\ t\right) \\ \eta_{t1} &= \frac{c_1}{c_1+c_2}H_i\cos\begin{pmatrix} k_2(x-x_1) \\ +k_1x_1-\omega\ t \end{pmatrix} \\ \eta_{r1} &= \frac{1}{2}\frac{c_1-c_2}{c_1+c_2}H_i\cos\begin{pmatrix} k_1x- \\ 2k_1x_1+\omega\ t \end{pmatrix} \end{aligned} \tag{10}$$

Now consider a second region boundary $x = x_2$, where $x_2 > x_1$. This is shown in Fig.2. The transmitted wave from the first step becomes the incident wave at the second step. The abrupt depth change at second step will result in reflected and transmitted waves η_{r2} and η_{t2}. These can be written as follows:

$$\begin{aligned} \eta_{t1} &= \frac{H_{t1}}{2}\cos\left(k_2x-\omega\ t+\varepsilon_{t1}\right) \\ \eta_{t2} &= \frac{H_{t2}}{2}\cos\left(k_3x-\omega\ t+\varepsilon_{t2}\right) \\ \eta_{r2} &= \frac{H_{r2}}{2}\cos\left(k_2x+\omega\ t+\varepsilon_{r2}\right) \end{aligned} \tag{11}$$

The procedure followed for the first step is repeated to solve the system of equations for the second step. This technique can be repeated for an arbitrary number of steps to obtain the general result:

$$\begin{aligned} H_{t(n)} &= 2\frac{c_n}{c_n+c_{n+1}}H_{t(n-1)} \\ H_{r(n)} &= \frac{c_n-c_{n+1}}{c_n+c_{n+1}}H_{t(n-1)} \end{aligned} \tag{12}$$

$$\begin{aligned} \varepsilon_{t(n)} &= \left(k_n-k_{n+1}\right)x_n+\varepsilon_{t(n-1)} \\ \varepsilon_{r(n)} &= -2\ k_nx_n-\varepsilon_{t(n-1)} \end{aligned} \tag{13}$$

If $(n-1)=0$, then $H_{t(n-1)} = H_i$ and $\varepsilon_{t(n-1)} = \varepsilon_i = 0$. Using (12) and (13), the general free surface equation can be written as Eq. (14):

$$\begin{aligned} \eta_{t(n)} &= \frac{H_{t(n)}}{2}\cos\left(k_{n+1}x-\omega\ t+\varepsilon_{t(n)}\right) \\ \eta_{r(n)} &= \frac{H_{r(n)}}{2}\cos\left(k_nx+\omega\ t+\varepsilon_{r(n)}\right) \end{aligned} \tag{14}$$

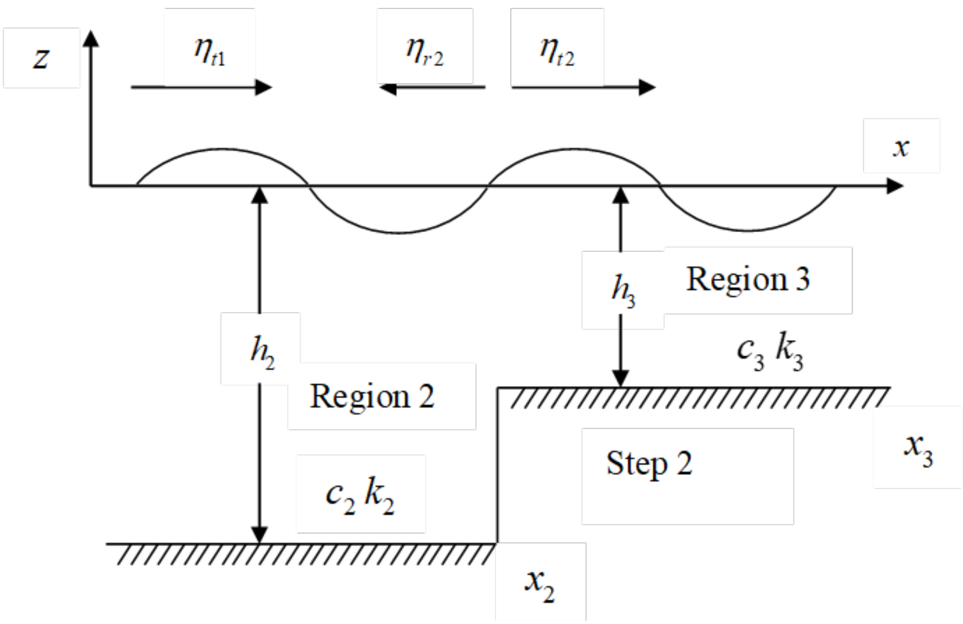

Fig.2 Sketch of second step.

3. RE-REFLECTED WAVE MODEL

Up to this point, the determination of reflected and transmitted waves from each step is rather straightforward. Unfortunately, re-reflection occurs between steps, and these are not negligible. An example of re-reflection is as follows. The incident wave propagates across the first step to give a transmitted wave η_{t1}. This wave is then partially reflected from the second step. This reflected wave then propagates back to the first step, where it is partially reflected again back to the second step. It is clear that this process develops many reflected waves. Fortunately, with each reflection, the importance of this mechanism decreases. This is because at each step there is an only partial reflection, so multiple reflections tend to get smaller with each re-reflection. To conserve energy, the re-reflected and retransmitted waves must be considered.

In Fig.3, η_{r3} is called the first-order reflected wave. The retransmitted and re-reflected waves are designated η_{r32} and η_{t32}, respectively. Waves resulting from retransmission and re-reflection of previously reflected waves are termed second-order waves. For second-order waves, subscripts *r* and *t* are used to denote transmitted and reflected waves, respectively. Thus, waves propagating in the positive *x*-direction are designated by “*t*” and those propagating in the negative *x*-direction are designated “*r*”, regardless of the order of the wave. The indices (*j*,*n*) describe a wave that is retransmitted and re-reflected from step *n* (at x_n), which previously originated in a reflection from step *j* (at x_j). The waves shown in Fig.4 can be defined as:

$$\eta_{r3} = \frac{H_{r3}}{2}\cos\left(k_3 x + \omega t + \varepsilon_{r3}\right)$$

$$\eta_{r32} = \frac{H_{r32}}{2}\cos\left(k_2 x + \omega t + \varepsilon_{r32}\right) \qquad (15)$$

$$\eta_{t32} = \frac{H_{t32}}{2}\cos\left(k_3 x - \omega t + \varepsilon_{t32}\right)$$

where η_{r3} is known? To solve these equations, boundary conditions (3) and (7) are applied. The second-order wave heights and phases can be written as:

$$H_{r32} = 2\frac{c_3}{c_2 + c_3} H_{r3}$$

$$H_{t32} = \frac{c_3 - c_2}{c_2 + c_3} H_{r3} \qquad (16)$$

$$\varepsilon_{r32} = \left(k_3 - k_2\right) x_2 + \varepsilon_{r3}$$

$$\varepsilon_{t32} = -2\, k_3 x_2 - \varepsilon_{r3} \qquad (17)$$

η_{r31} and η_{t31} are written as:

$$\eta_{r31} = \frac{H_{r31}}{2}\cos\left(k_1 x + \omega t + \varepsilon_{r31}\right)$$

$$\eta_{t31} = \frac{H_{t31}}{2}\cos\left(k_2 x - \omega t + \varepsilon_{t31}\right) \qquad (18)$$

Applying the boundary conditions, the unknowns can be written as:

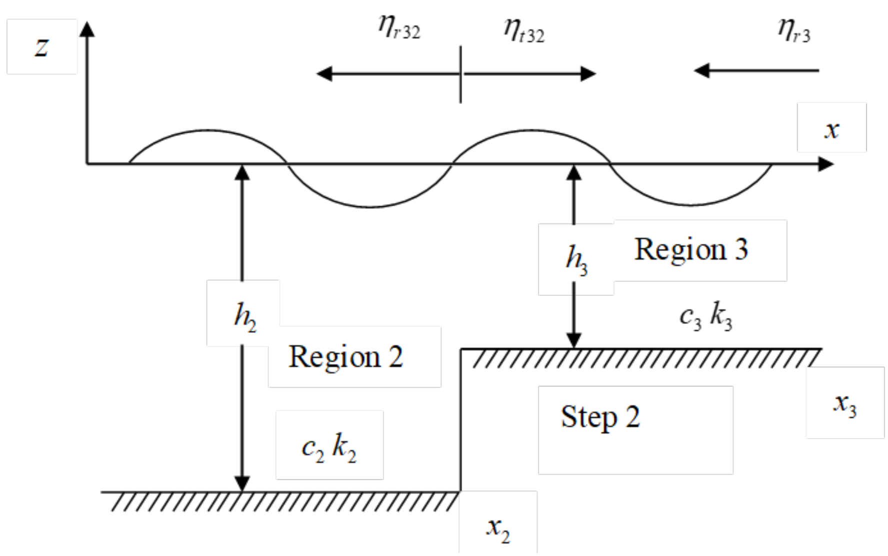

Fig.3 An example of retransmission and re-reflection of the waves.

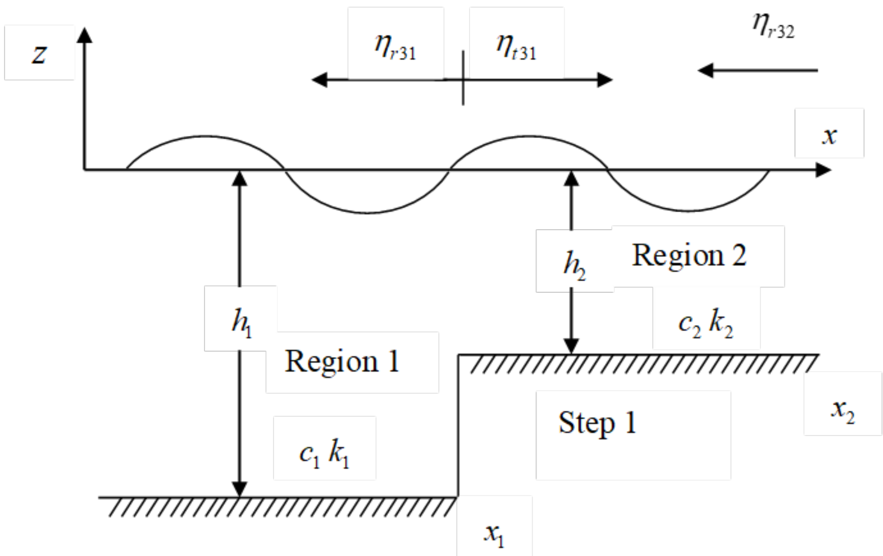

Fig.4 η_{r32} is transmitted and reflected over $x = x_1$. It becomes η_{r31} and η_{t31}.

$$H_{r31} = 2\frac{c_2}{c_1+c_2}H_{r32}$$
$$H_{t31} = \frac{c_2-c_1}{c_1+c_2}H_{r32} \tag{19}$$

and,

$$\varepsilon_{r31} = (k_2 - k_1)x_1 + \varepsilon_{r32}$$
$$\varepsilon_{t31} = -2\,k_2 x_1 - \varepsilon_{r32} \tag{20}$$

The repetitive formulation to get the second-order wave heights and phases can be written as:

$$H_{r(j,n)} = 2\frac{c_{n+1}}{c_n+c_{n+1}}H_{r(j,n+1)}$$
$$H_{t(j,n)} = \frac{c_{n+1}-c_n}{c_n+c_{n+1}}H_{r(j,n+1)} \tag{21}$$
$$\varepsilon_{r(j,n)} = (k_{n+1}-k_n)x_n + \varepsilon_{r(j,n+1)}$$
$$\varepsilon_{t(j,n)} = -2\,k_{n+1}x_n - \varepsilon_{r(j,n+1)}$$

The above repetitive formulations are valid for $n < j$. The case where $n = j$ is described by (12), (13), and (14). The complete secondary wave equations are:

$$\eta_{r(j,n)} = \frac{H_{r(j,n)}}{2}\cos\left(k_n x + \omega\, t + \varepsilon_{r(j,n)}\right)$$
$$\eta_{t(j,n)} = \frac{H_{t(j,n)}}{2}\cos\left(k_{n+1}x - \omega\, t + \varepsilon_{t(j,n)}\right) \tag{22}$$

The superposition of all waves originating from step n is:

$$\eta = \eta_{t(n)} + \eta_{r(n+1)} + \sum_{m=n+1}^{j}\eta_{t(m,n)} + \sum_{m=n+2}^{j}\eta_{r(m,n+1)} \tag{23}$$

where j denotes the final step. Figure 5 shows the waves over the nth step.

The reflection coefficient for the total system is calculated using the wave envelope method in region 1. For this purpose, the width of region 1 is extended by several wavelengths. The wave envelope is obtained by taking the maximum and minimum of the total waves at each location in region 1 with respect to time. The reflection coefficient is:

$$K_r = \frac{\eta_{max} - \eta_{min}}{\eta_{max} + \eta_{min}} \quad \text{in Region 1} \tag{24}$$

where η_{max} and η_{min} are the maximum and minimum of the wave envelope in region 1. Figure 6a shows the reflection coefficient calculated for an abrupt transition containing two steps using the wave envelope method over the slope *m* between 0.001 and 1. From the figure, the waves have to be reflected and transmitted a minimum of nine times to obtain a reflection coefficient near to 1.0. A calculation including the effects of depth-limited breaking waves was also done. The transmitted wave height $H_{t(n)}$ is subjected to wave breaking conditions [5].

$$H_{t(n)} = 0.8\,h_n \tag{25}$$

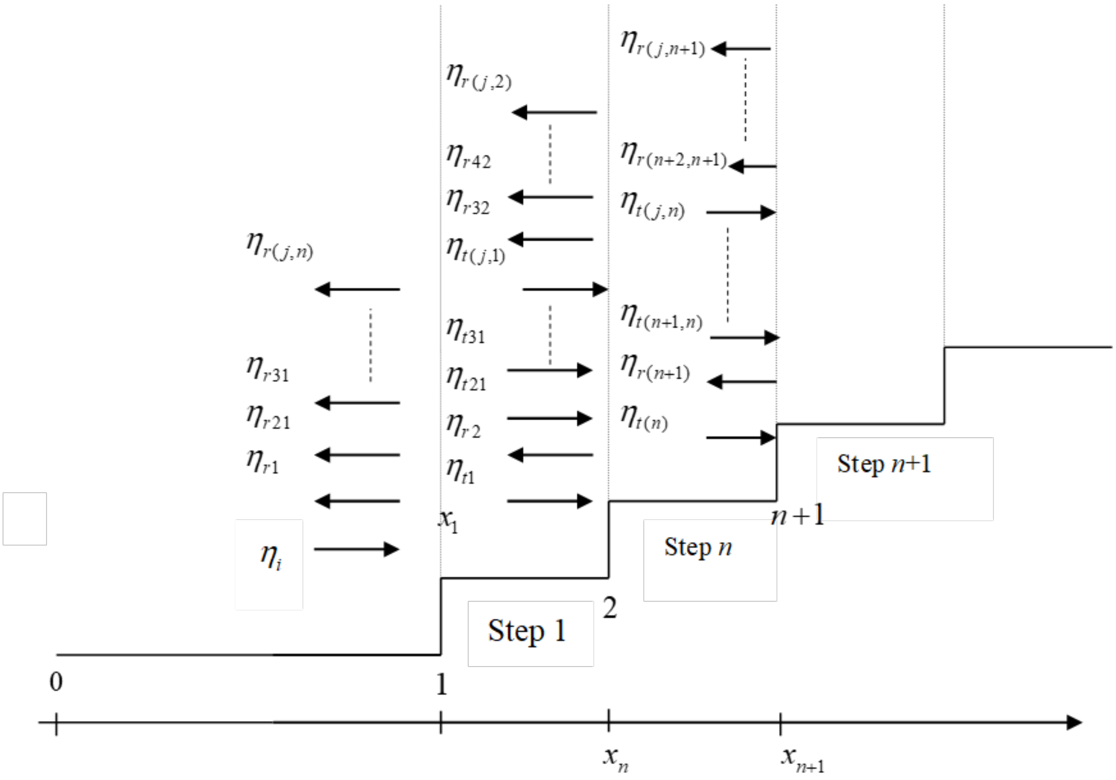

Fig.5 Summary of waves over step *n*.

The results are summarized in Fig.6b. Breaking waves reduce the reflection coefficient by about 30%. The average K_r is around 0.7. More steps would yield a more realistic reflection coefficient. Figure 8 shows that the total energy of the system is conserved when the waves are re-reflected and retransmitted more than four times, with the reflection coefficient approaching one.

Figure 9 shows that the reflection coefficient decreases for beaches with smaller grain sizes. Waves dissipate more energy on beaches with smaller grain sizes. The slope is milder and the waves break farther offshore. Thus, wave energy dissipates over a larger area. A beach with a small grain size and mild slope is called a dissipative beach. When the grain sizes are larger, the beach is steeper, and the waves break at a location closer to the shore. Energy is dissipated over a limited area on the steep beach. A beach with larger grain sizes is called a reflective beach.

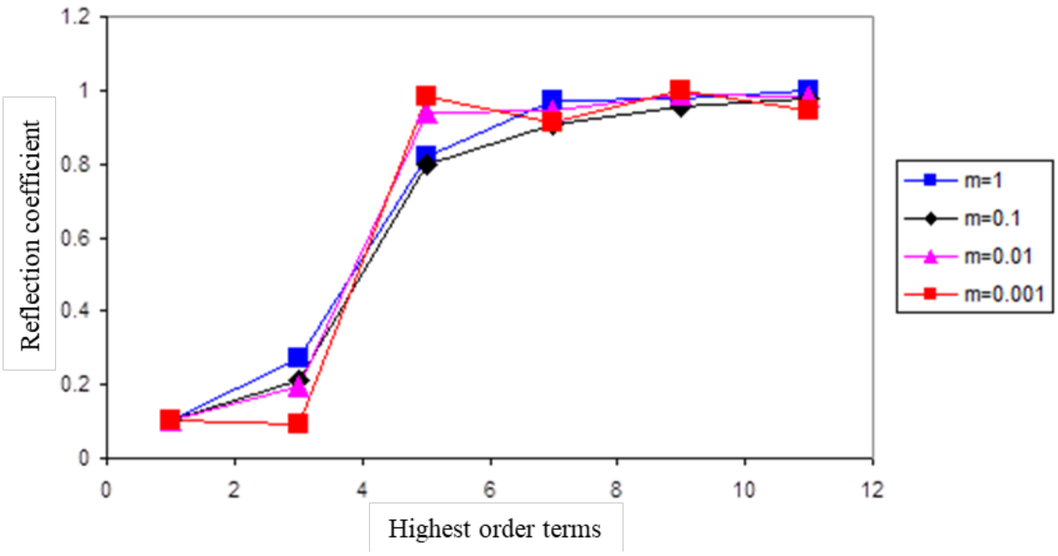

Fig.6a Reflection coefficient K_r over a two-step abrupt transition with different slopes without breaking waves.

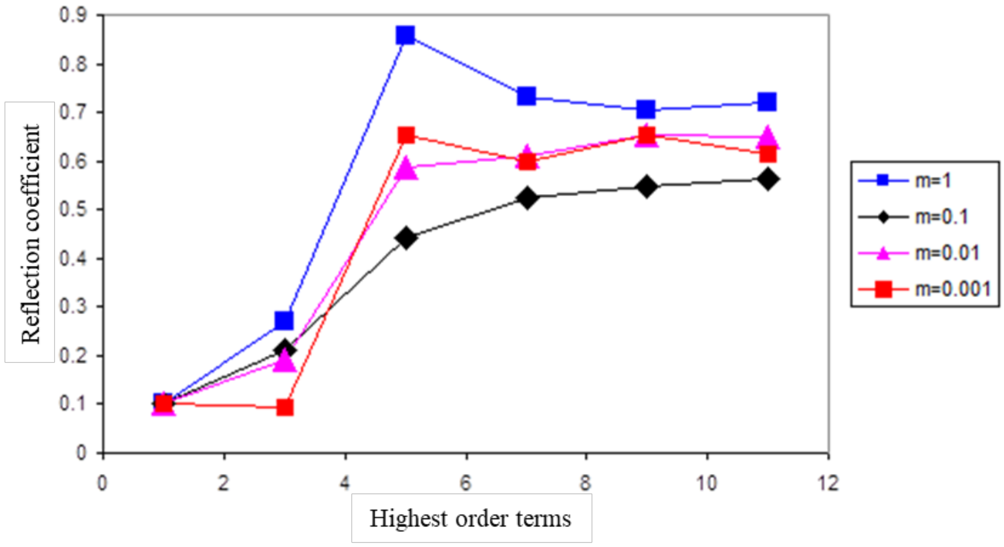

Fig.6b Reflection coefficient K_r over a two-step abrupt transition with different slopes with breaking waves.

4. DISCUSSION AND COMPARISON WITH OTHER RESEARCH

The results of the current research were compared with the result obtained by Nagashima (1971) [10]. He ran a physical model in a wave-flume 3.3 m long, 30 cm deep, and 8 cm wide. The slope was varied from 3/50 to vertical. Several harmonic waves with wave steepness of 1.8% were modeled. Mei (1989) also gives an empirical formula to calculate the reflection coefficient as a function of the surf parameter ξ [11] . The formula can be written as:

$$K_r = 0.1\ \xi^2 \tag{26}$$

The formula is obtained by best fitting data from Battjes (1974) and is valid for mild slopes [12]. Zanuttigh *et al.* (2008) also give a simple empirical formula relating the reflection coefficient with the surf parameter for impermeable rock slopes [13], and the best fit is given by:

$$K_r = 0.17\ \xi^{0.7} \tag{27}$$

Figure 10 shows the comparisons. The current research shows a significant difference for a small slope, where the abrupt transition is introduced into the system. It seems that the abrupt change leads to significant reflection waves in comparison to smooth slopes, as demonstrated by other researchers.

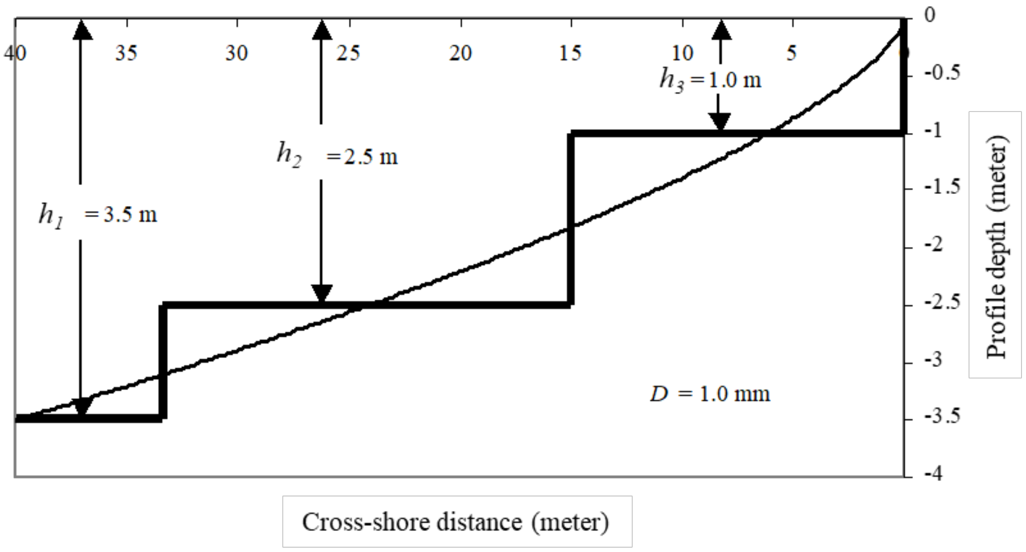

Fig.7 Best fit of 2 steps to the Dean beach profile with D = 1.0 mm.

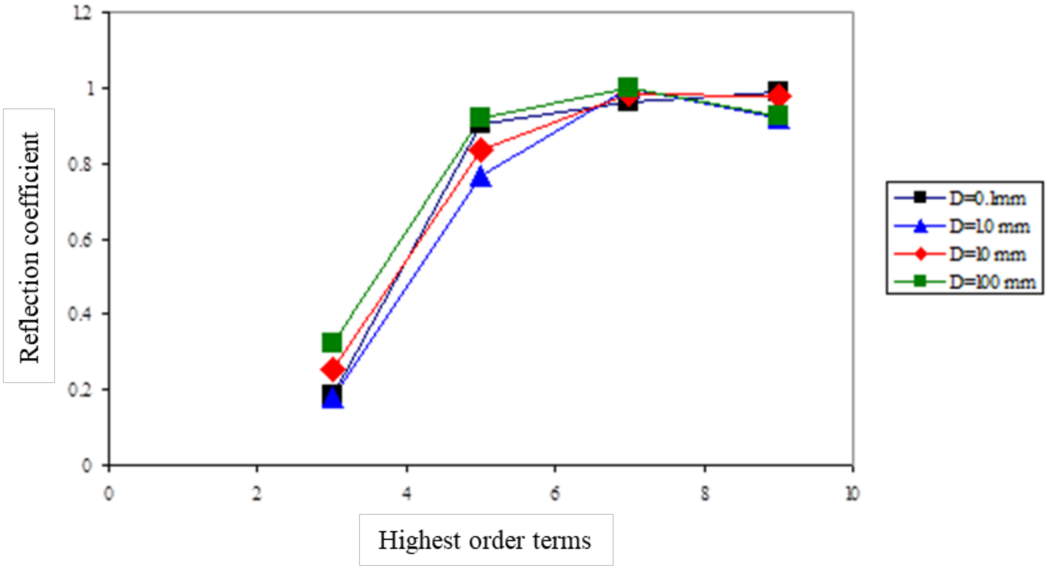

Fig.8 Reflection coefficient for Dean beach profile without breaking waves.

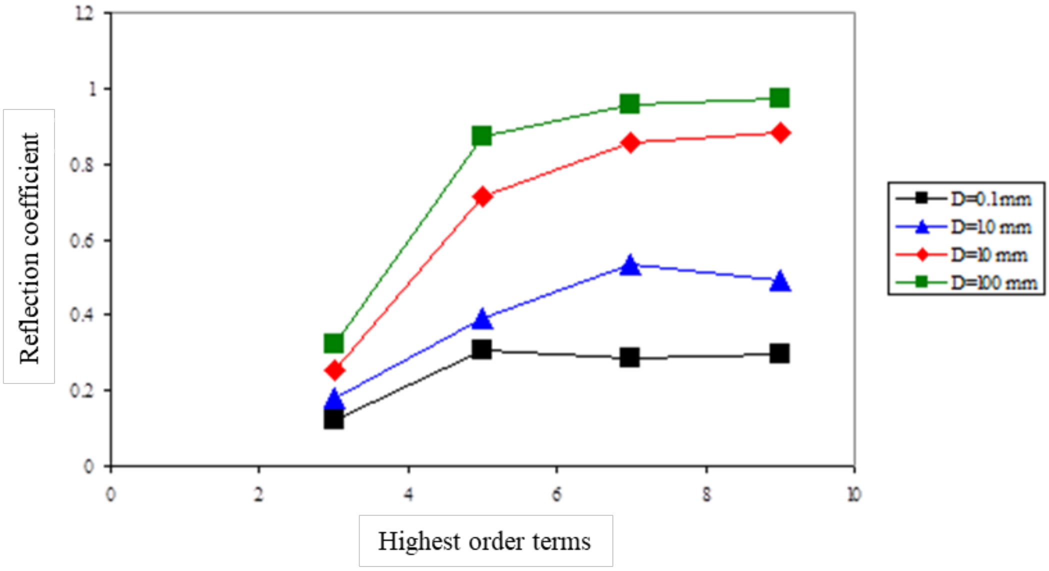

Fig.9 Reflection coefficient for Dean beach profile with breaking waves.

5. CONCLUSION

A simple long-wave reflection and transmission model has been presented. The model includes multiple steps and depth-limited breaking waves. For cases of abrupt transition featuring a Dean beach profile with different slopes and beach, materials are also demonstrated. The waves are re-reflected and retransmitted several times. The total energy of the system is conserved when the waves are re-reflected and retransmitted more than five times.

The wave breaking causes the reflection coefficient to decrease by about 30% since parts of the wave energy dissipate on the sloping beach. The average reflection coefficient for the breaking-waves condition is 0.7. From the comparisons in Fig.10, it is found that for a small slope, the abrupt depth changes give up to 60% more significant reflected waves than the smooth slope.

6. ACKNOWLEDGMENTS

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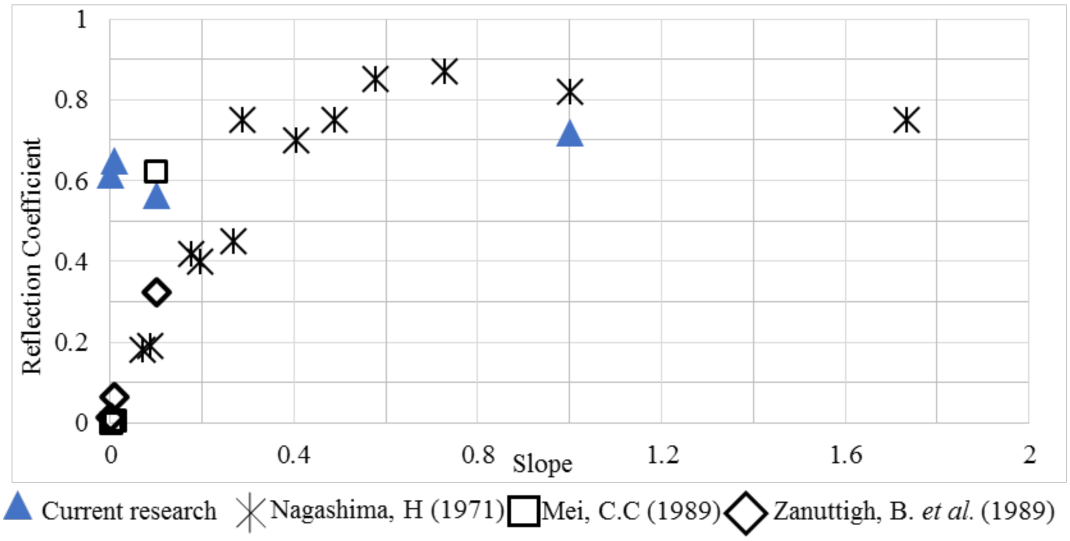

Fig.10 Comparisons of reflection coefficient with other researches.

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