

WATER MOVEMENT AND DEFORMATION IN UNSATURATED MULTI-LAYERED SLOPE UNDER HEAVY RAINFALL

Tang Junfeng¹, Uchimura Taro¹, *Tao Shangning¹, Huang Dong³, Xie Jiren²

¹Department of Civil & Environmental Engineering, Saitama University, Japan;

²Department of Civil & Environmental Engineering, University of Tokyo, Tokyo, 113-8654, Japan;

³Institute of Mountain Hazards and Environment, Chinese Academy of Science, China

*Corresponding Author, Received: 16 March 2019, Revised: 22 Nov. 2019, Accepted: 17 March 2020

ABSTRACT: In 2013, a number of shallow landslides triggered by heavy rainfall affected a mountainous area which located on Izu-Oshima island (Eastern Japan). These slopes are consist of fine soil layers and coarse soil layers which have a different permeability coefficient and soil water characteristic curves. To clarify the characteristics of water infiltration in such unsaturated multi-layered slope and to assess the influence of the water content distribution on slope failure, 4 column tests and 2 groups of inclined slope model experiments were conducted. Silica No 1 ($D_{50}=3.10\text{ mm}$) and silica No 7 ($D_{50}=0.16\text{ mm}$) were used as the slope materials. The results indicated that advancing wet front apparently stopped at the interface between the silica No 7 layer and silica No 1 since the capillary barrier works and then lateral water flow occurred along with the interface and infiltrate into next layer when soil approaches saturation. It was found that when lower water content θ in fine and coarse layer, unsaturated permeability coefficient K in coarse layer ($K=5.54\text{E-}06\text{ cm/s}$) is smaller than fine layer ($K=1.08\text{E-}04\text{ cm/s}$) since matric suction is 2.5 kPa, which result that capillary barrier works. In addition, higher water zone still exists at the bottom of the fine layer after drainage for a long period which results in the failure in this zone firstly when second rainfall was applied. Piping occurred at the bottom of the model when amount of water exists inside the slope. Although toe of slope reach to saturated condition is later in multi-layer slope, failure time is earlier.

Keywords: *Unsaturated soil, Water infiltration, Capillary barrier, Permeability coefficient, Multi-layer slope, soil Piping*

1. INTRODUCTION

This study deal with a slope consist of fine layer and coarse layer. It has a deep influence on the process of rainfall water infiltration and volume water content distribution in slope, thus governing the failure of slope. These kinds of multilayer slopes are present in many landscapes. For example, the catastrophic landslides that occurred in May 1998, involved pyroclastic covers laying upon the slopes of Pizzo D'Alvano, in Campania Southern Italy [1]. Another example is the multi-layer slope consists of volcanic sand and loess in Japan, which have the failure on the surface in Izu-Oshima in 2013. Rainfall infiltrating water owing to the build-up of capillary barriers [2] that occur at the interface between unsaturated fine and coarser soil layers. Capillary barriers can maintain a high degree of saturation in the soil above them which result into a different failure part in multi-layer slope [3]. These phenomena are related to capillary tension that limits the downward movement of water from a finer soil into underlying coarser soil. In some situations, capillary forces in the upper fine layer can no longer hold water, any additional infiltration is transmitted to the lower coarse layer. This situation happens at a critical distance from the top of the slope that can be estimated using a model proposed by Ross [4].

In this paper, building up and breakthrough point

of the capillary barrier has been taken into consideration and its influences on slope failure time and failure modes are discussed based on column tests, inclined model slope experiments and mathematic calculation.

The main objectives of this study are to evaluate the influence of capillary barrier and its diversion capacity on slope failure and the different failure modes between single layer and multi layer slope. SWCC tests of fine sand and coarse sand were also conducted to estimating the different hydraulic conductivities to explain how the capillary barrier works.

2. TESTING MATERIALS AND METHODS

Two soils from Japan, Silica No 1 and Silica No 7 were used in the lab experiments, which were excavated from Tono-Area Gifu Province. The sieve tests were conducted using the JGS Geotechnical Society standard test methods (JGS0131-2009). The details of the experiment properties are shown in Fig 1. The dry density of the coarse layer is 1.43 g/cm^3 while that of the fine layer is 1.33 g/cm^3 . In addition, Table 2 and Fig 5 show soil water characteristic curves of silica No 1 & 7 in both drying and wetting process and the fitting parameters by the VG model. It was found that saturated water content and residual water content in silica No 1 is lower than silica No 7, and volume water content is also lower under the

same matric suction condition.

The other basic physical properties of test materials, including specific gravity G_s , maximum dry density, soil grain size distribution was measured in accordance with JGS standard test methods and the result are shown in Table 1. According to the soil classification system (USCS), Silica No 1 is classified as the coarse sand while No7 is fine sand.

Table 1 Basic properties of Silica No 1 and No 7

Description	Silica No 7	Silica No 1
Specific gravity G_s	2.63	2.62
Gravel content (>4.75mm; %)	0	97.13
Sand content (%)	87.31	2.86
Fines content (<0.075mm; %)	11.64	0
D_{10} (mm)	0.043	2.26
D_{50} (mm)	0.152	3.52
D_{60} (mm)	0.165	4.21
Maximum dry density (g/cm^3)	1.556	1.52
Minimum dry density (g/cm^3)	1.271	1.38

3. SWCC AND UNSATURATED HYDRAULIC CONDUCTIVITY OF TESTING MATERIALS

SWCC and unsaturated hydraulic conductivity of sands was obtained in lab. Fig.1 shows the relationship between matric suction and volume water content. Both drying process and wetting process experiments were conducted showing a difference in both sands. In this case, hydraulic conductivity increased with water content increase in coarse and fine sand.

The soil-water characteristic curves of the soil have been modeled with the van Genuchten-Mualem model [5], namely:

$$S_e = \left\{ 1 + (-\alpha h)^n \right\}^{-m} \quad (1)$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

$$m = 1 - \frac{1}{n}$$

In the above equation, the water retention curve has been expressed in terms of effective degree saturation. Where θ is the volumetric water content; θ_r and θ_s indicate residual and saturated values of the water content; a , m and n are the fitting parameters. h is the matric suction. a is a scaling parameter (units of m^{-1}) and the exponents n and m are parameters that

determine the shape of the retention curve. The hydraulic parameters are given in Table 2.

Table 2 Hydraulic properties of the silica No 1 and silica No 7

Description	Symbol (unit)	Soil	
		No 7	No 1
Drying curve			
Saturated volume water content	θ_s	0.44	0.42
Air-entry value	ψ_a (kPa)	2.44	0.62
Residual volume water content	θ_r	0.12	0.05
van Genuchten model	a	0.41	1.59
Fitting parameter	n	4.07	3.11
	m	1.42	0.67
Wetting curve			
van Genuchten model	a	0.42	0.03
Fitting parameter	n	4.51	1.42
	m	0.78	0.29

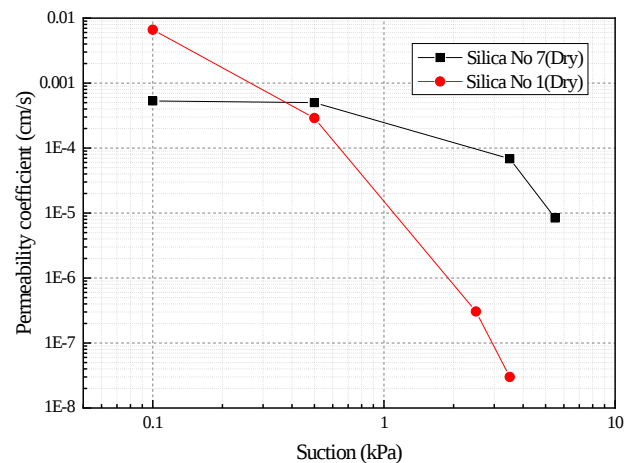


Fig. 1 Hydraulic conductivity of Silica No1 & No 7 in drying process and wetting process

The unsaturated hydraulic properties were measured by the variable head method (ASTM 2006) D2434-68. Fig. 1 shows that the volume water content in silica No 7 is higher than No 1 when under the same suction, which could explain how the capillary barrier works. Suction in fine layer decrease as the water content increase, capillary will breakthrough and rainwater start to infiltrate into the next layer. The obtained values range between 10^{-2} and 10^{-7} cm/s under different suction conditions. The results show that the hydraulic conductivity of the coarse sand is larger than that of the fine sand at almost saturation, while it is significantly smaller when the soil is unsaturated.

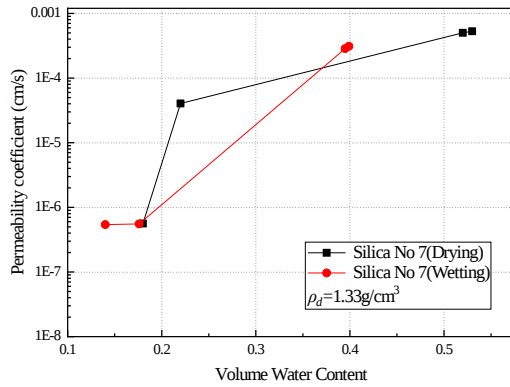


Fig. 2 Relationship between permeability coefficient and volume water content of Silica No7 in both Drying and Wetting process

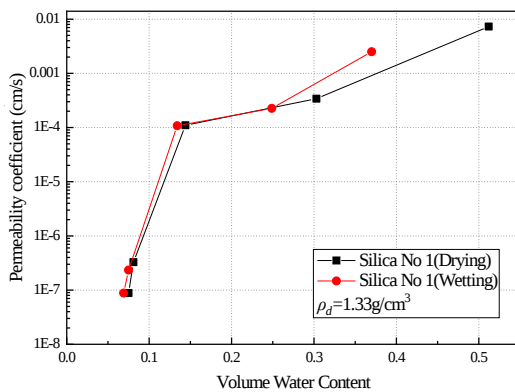


Fig. 3 Relationship between permeability coefficient and volume water content of Silica No1 in both Drying and Wetting process

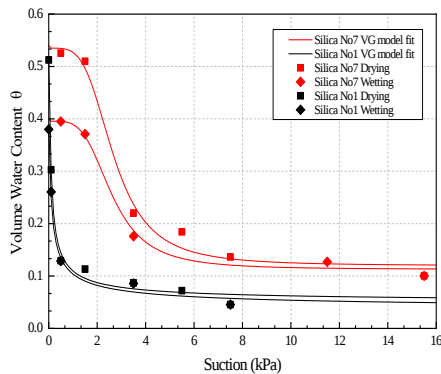


Fig. 4 Soil water characteristic curves of silica No 1 & 7 in both drying and wetting process and the fitting parameters by the VG model

4. MODEL EXPERIMENTS

3.1 Column Infiltration Tests in Layered Soil

A series of laboratory column infiltration tests have been performed in the laboratory to observe and to analyze the capillarity barrier behavior at the laboratory scale. The apparatus consisted of the

transparent column, whose diameter is 155mm and height is 450mm. In the experiments, moisture sensors (METRE, EC-5) were placed, at various locations, at four different depths within the three soil layers (at 10 cm, 15 cm, 22.5 cm and 30 cm below the top surface, respectively) in test I and II while 5 different depths in test III and IV (at 10 cm, 15 cm, 22.5 cm and 30 cm below the top surface, respectively). HOBO-loggers were used to get the data of water content every 10 seconds. The density of coarse layer and the fine layer are 1.43 g/cm³ and 1.33 g/cm³ while the initial water content is 5%. Results from four sets of tests are reported on Fig. 6. In the column Test I, II, III and IV.

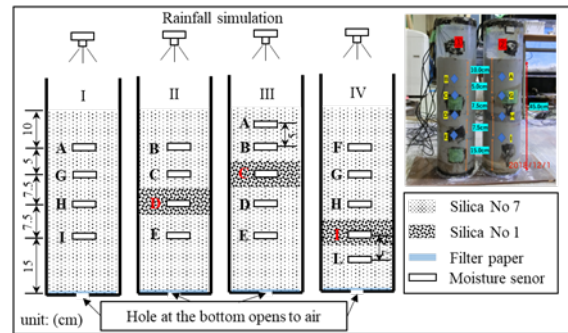


Fig. 5 Schematic view and photo of the experimental apparatus for column Test I, II, III and IV

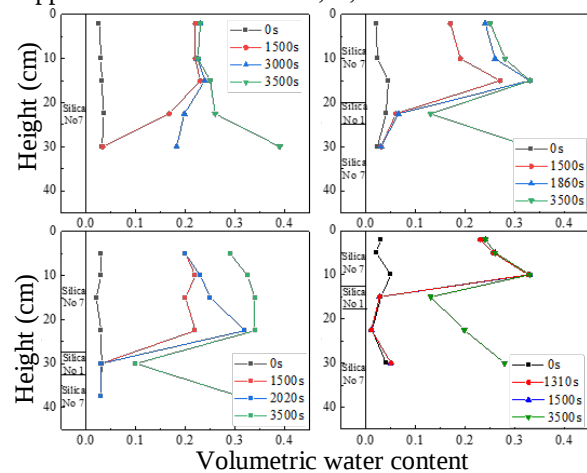


Fig. 6 Profiles of measured volumetric water content in test I, II, III, and IV

3.1.1 Test procedures

The sand was packed into the cylinder and hammered to get the designed density in 5cm increments. During the wetting experiments, all of the column tests I, II, III and IV (Fig.5) has been subjected to a rainfall intensity of 75mm/h for about 2 hours (rainfall stage) until the attainment of a nearly steady state condition. And to a water drainage process of about 72 hours (dry stage). The bottom of the column has a hole (diameter = 35mm) covered with filter paper, which is open to the atmosphere.

3.1.2 Test results and analysis

Fig. 6 shows the progress of the infiltrate wetting front to a different depth. The advancement in the beginning of rain for 4 tests of the wetting front was similar for all cases because they had the same soil types in the upper soil layer. The difference occurred at the interface of the fine layer and coarse. For example, in tests I and II, the arrived time of water at the depth of 100 mm and 150mm are 510 seconds and 890 seconds respectively. However, the arrived time at central coarse layer (depth of 225 mm) were 1380 seconds and 1890 seconds respectively, which showed a time delay around 13minutes at this area. These results show that the progress of the wet front apparently stopped at the interface between the silica No 1 & No 7 when fine layer overlying the coarse layer since capillary barrier worked. In tests III and IV, the same phenomenon could also be observed at the interface when the depth of upper fine layers were different.

In addition, time histories of the volume water content of Test I, II, III and IV (in Wetting Process) at different depths are illustrated in Fig. 7. Taking the original data of Test I as an example (Fig. 7 I), Point B, C, D and E showed a sharp increase of soil water content when the wetting front arrived, which soon reached a stable and nearly saturated value. The probe D and E were higher than that at Probes B and C. This indicated that ponded infiltration had caused sealed air and result in an unsaturated condition in the top soil layer while it was not so obvious in the bottom layer. For Probes E in column, the obvious increase of soil water content not only occurred when the wetting front arrived but also after the wetting front had reached the bottom of the soil column (at about 56 min).

The reason is that the wetting front stopped for a while above the interface between the silica No 7 (which was fine layer) and silica No 1 (which was coarse layer) while rainfall on the surface did not stop and rainfall water accumulated. Therefore, water started accumulating from the bottom and upwardly rewetting the sand.

As a result, the bottom of the upper fine layer had a higher water content in the multi-layered model (Tests II, III, IV) while it is lower in single layer test (Tests I). It means that water accumulation at the interface results a higher water content zone above the coarse layer in the multi-layer test. Compared Test III with IV, the area of higher water content zone became larger while the thickness of the fine layer overlying the coarse layer is larger.

5. MODEL TESTS OF THE MULTI-LAYERED SLOPE UNDER RAINFALL

To evaluate the effect of capillary barrier acting on a slope resembling, the experiments of multi-layered slope have been set up. The apparatus for the physical model experiments consisted of an inclined steel box, a rainfall simulator, water content sensors and in soil (Fig. 8 b). Details pertaining to each subsystem are as follows: (i) the inclined steel box was 1.0 m long, 0.3m wide, and 0.5m high; (ii) the sidewalls of the box were made of acrylic plate to visually observe the advance of the wetting front and failure process during rainfall infiltration; (iii) pictures were taken every 30 seconds by cameras (Fig. 8).

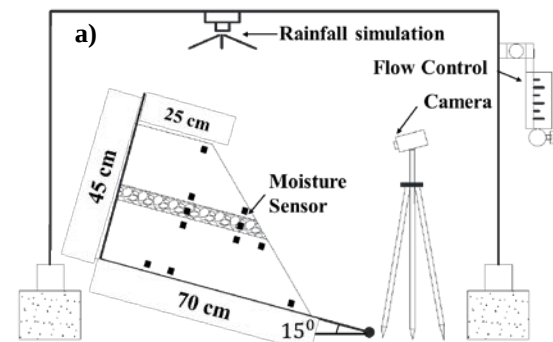


Fig. 8 a) Schematic diagram of the experimental apparatus for multi-layered slope under rainfall: side-view of the multi-layered slope

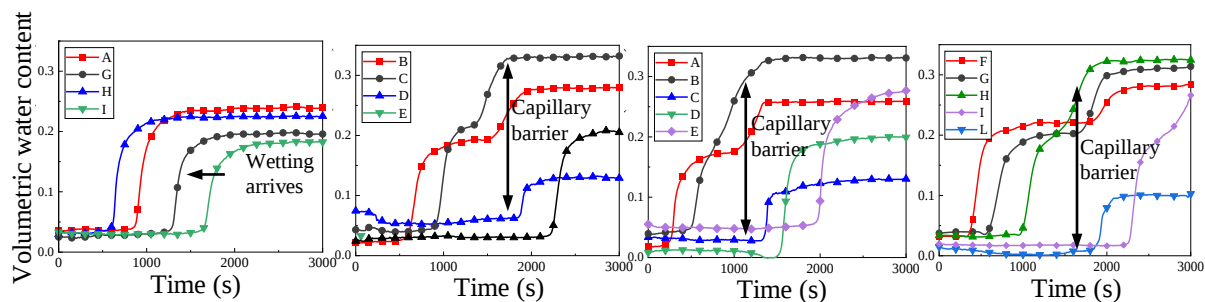


Fig.7 Variation in water content versus time in different observation points in the column tests. I) single layer column; II) multi-layer column; III) upper multi-layer column; IV) lower multi-layer column.

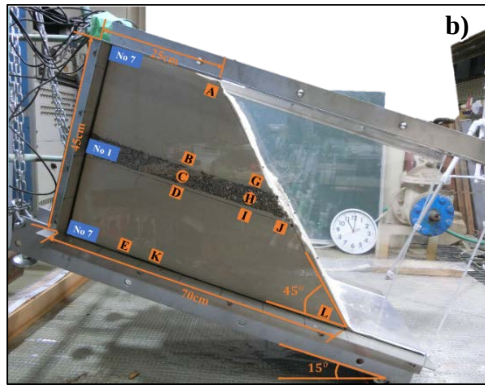


Fig. 8 b) Schematic diagram of the experimental apparatus for multi-layered slope under rainfall: apparatus for the physical model experiment

5.1.1 Testing procedure

Soil preparation. The silica No 1 and silica No 7 used to make the slope was dried in an oven for 48 hours. Then, the amount of water was added to the soil to achieve the initial water content.

Compaction of soils. Slope model was constructed in the model box when the soil has been prepared. The prepared soil was compacted and placed in a series of horizontal layers. Silica No 7 and No 1 were placed into the model box in layers and compacted to achieve the dry density of 1.33 g/cm^3 and 1.43 g/cm^3 , respectively. Each layer was tamped equally rod to a thickness of 5cm and repeat the procedures until the height of slope was achieved. Fig. 8 a) shows the information about slope completed. Layers of Silica No 1 was placed as the coarse layer in multi-layer slope.

Positions of sensors and cameras. During the soil placement, moisture sensors were placed at the specific locations within the slope and time of recording the quantity of water content was 10 seconds. Cameras were put in 6 different locations to record the failure condition of the slope every 30 seconds. And 6 black dots were inserted laterally into the surface of the slope to as the marks.

Rainfall Simulation. When the initial readings of all sensors and camera were stable, the rainfall simulation commenced. A rainfall sprinkler (Fig. 8)

was used to simulate artificial rainfall at different intensities from 45 mm/h to 85 mm/h . During the first rainfall, 4 hours was applied with 45 mm/h . Then the slope model was subjected to a drying process without rainfall; The second rainfall was applied for 4 hours with intensity was 45 mm/h . The experiment was stopped with the assumption that seepage had reached a steady-state condition.

Table 3 General information for the slope model experiment

	Group 1		Group 2	
Inclined degree	0		15	
Initial water content	5%		5%	
Unit weight of dry soil (g/cm^3)	No 1	14.4	No 1	14.4
	No 7	13.3	No 7	13.3
Rain 1 Intensity	45 mm/h		45 mm/h	
Rain 1 Duration	4 h		4 h	
Drying Period 1	72 h		72 h	
Rain 2 Intensity	45 mm/h		45 mm/h	
Rain 2 Duration	4 h		4 h	
Drying Period 2	72 h		72 h	

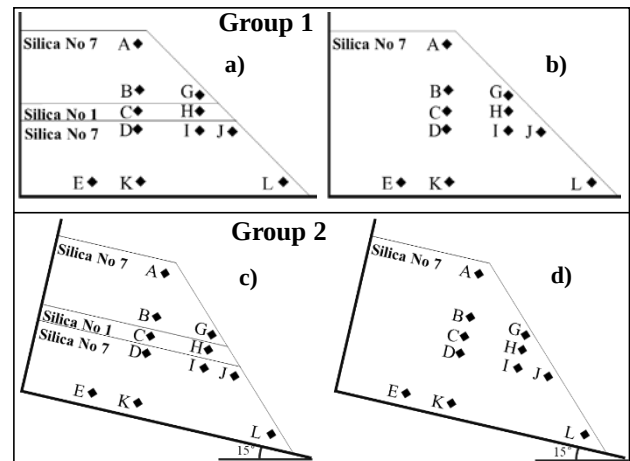


Fig. 9 Schematic diagram of the experimental apparatus for different groups of slopes under rainfall

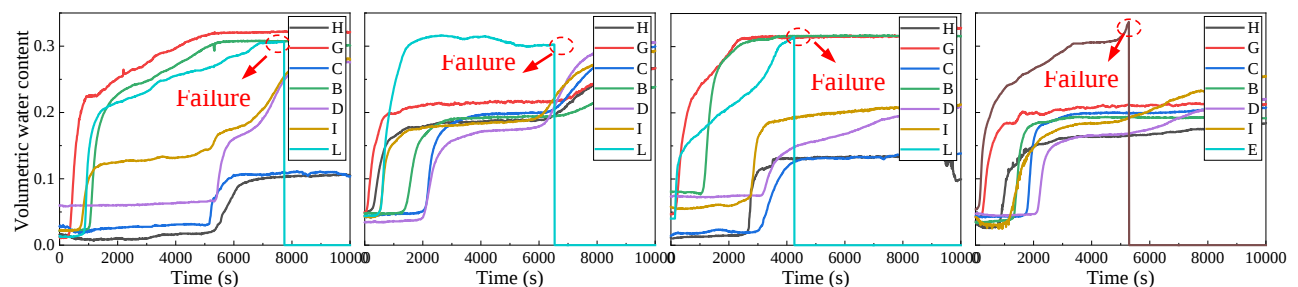


Fig. 10 Water content variation with time at different locations in the flat and inclined groups slopes. a) flat multi-layer slope; b) flat single layer slope; c) inclined multi-layer slope; d) inclined single layer slope.

5.1.2 Experiment result and discussion

In Fig. 10, water content decreases sharply which means the failure occurred at this point since the separation of sensors and soil. Comparing the water content at point L in the flat group (Fig. 10 a and b), point L reached to the saturated condition later in multi-layer since capillary barrier prevents the water infiltrate into the bottom which makes the slope more stable and caused a time delay in failure time. Comparing the water content at point L in the inclined group (Fig. 10 c and d), point L reached to saturated condition still later but the failure time is earlier (about 20 minutes) than single layer slope which means the multi-layer slope is more dangerous under rainfall situation.

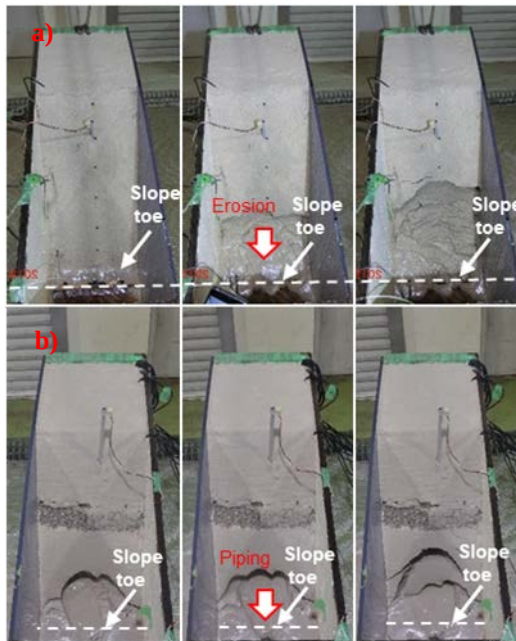


Fig. 11 Different failure modes occurred in the toe of slope under rainfall condition. a) sliding from toe in inclined single layer slope; b) piping occurred at the bottom in inclined multi-layer slope.

From Fig.11, different failure modes were observed during the slope experiments. Sliding occurred from the toe of slope (Fig. 11 a) gradually in single layer slope when the rainfall was applied. In the inclined multi-layer slope, piping occurred at the bottom of the slope. Piping (Fig. 11 b) occurred at the bottom when amount of water exists in inside the slope. Although point L reach to saturated condition is later in multi-layer slope, a failure time is earlier.

In order to clear the water movement across the interface, more sensors were put in the coarse layer (Point H, M, C, N and O) in inclined multi-layer slope experiment, 1st and 2nd rainfall were also applied which durations were 20 minutes and 50 minutes respectively.

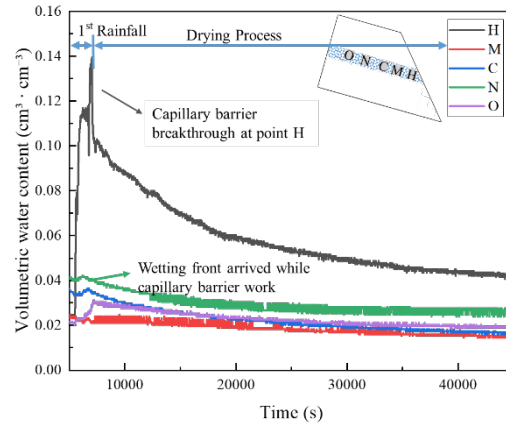


Fig. 12 Volume water content variation with time at different locations along the coarse layer during the first rainfall (20 minutes)

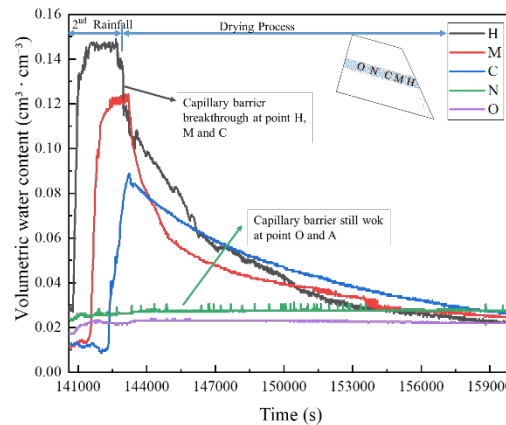


Fig. 13 Volume water content variation with time at different locations along the coarse layer during the second rainfall (70 minutes)

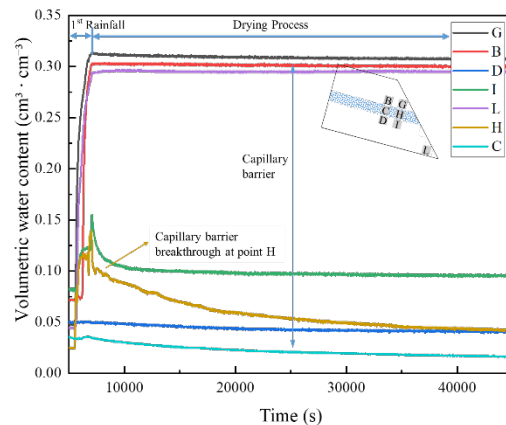


Fig. 14 Comparison between water content in different layers during the first rainfall (20 minutes)

Fig. 15 Comparison between water content sensors in different layers during the second rainfall (70 minutes)

Fig. 12 (case c) shows the water content at point H, M, C, N, and O which at different locations in the coarse layer. The graph shows that when 1st rainfall was applied, the water content at point O and M

increased firstly since the location was closest to surface, then N, C and M increased when wetting front arrived the interface and have a small flux before capillary barrier breakthrough. Meanwhile, water content at point H increased quickly and showed a higher water content in Silica No 1 since the capillary barrier didn't work. When rainfall was stopped and the dry process started, water drained down easily with a rapid reduction in water content in the coarse layer. Similar behavior was also observed when the second rainfall was applied. It was found that the water content in point H, M and C increased gradually with the rainfall continued while water content at point O and N were stable (Fig. 13).

Fig. 14 shows the water content history of point B and G (above the coarse-grained layer), C and H (in the coarse layer) D and I (below the coarse layer) which were put in three different soil layers. This graph shows that water content above the coarse layer (point B and G) remain wetter than the soil below the coarse layer (point D and I). The sand above coarse layer showed 30% of volume water content all the time, suggesting that water didn't drain down through the coarse layer when first rainfall was stopped while the below showed a sharp reduction in the dry process.

6. DISCUSSION

The diversion capacity is the maximum flow that a capillary barrier can divert water flux and the diversion length is the horizontal length from the top of the slope to the breakthrough. In order to evaluate the diversion length of a capillary barrier, following assumptions are made: (1) suction profile in the fine layer could be estimated using the linear method [6]; (2) upper fine layers are thick enough; (3) the interface is inclined and longer than the diversion length; (4) continuous rainfall is applied on the top of the slope.

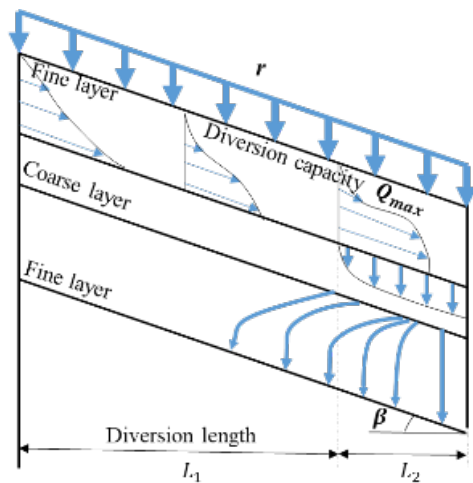


Fig. 15 Calculation of diversion length and capacity in an inclined slope

Based on these situations [4], diversion capacity and diversion length of capillary could be calculated, as the equation:

$$Q_{max} = \int_{z_{0c}}^{z_{conc}} v_h(z) dz \quad (2)$$

Where, Q_{max} is the maximum unsaturated lateral diversion capacity (cm^2/s); $v_h(z)$ - velocity profile as the function of elevation (cm^2/s); z_{0c} - elevation of base of the upper fine layer (cm); z_{conc} - elevation where the maximum suction in the upper coarse layer (cm);

According to the linear method, elevation can be transformed into suction, and matric suction ψ at the contact interface between two layers must be continuous across the boundary:

$$\begin{aligned} z_{0f} &= z_{conc} \\ \psi_{0f} &= \psi_{maxc} \end{aligned} \quad (3)$$

According to the equation (2) and (3), equation (4) can be obtained and can be deducted via Darcy's law.

$$Q_{max} = \int_{\psi_{conc}}^{\psi_{conf}} v_h(\psi) d\psi = \int_{\psi_{conc}}^{\psi_{conf}} i_h k(\psi) d\psi \quad (4)$$

Where, i_h - the lateral hydraulic gradient, which equals to the tangent of slope $\tan \beta$; $k(\psi)$ - k function; β -inclined angle of slope($^\circ$).

The maximum lateral diversion flux is

$$\begin{cases} Q_{max} = k_{sat} \tan \beta \int_{\psi_{conc}}^{\psi_{conf}} k_{rw}(\psi) d\psi \\ k_{rw}(\psi) = \frac{\left\{ 1 - (\alpha \psi)^{mn} \left[1 + (\alpha \psi)^n \right]^{-m} \right\}^2}{\left[1 + (\alpha \psi)^n \right]^{m/2}} \end{cases} \quad (5)$$

According to the formulas (5), the lateral diversion capacity of the inclined capillary barrier can be diverted down dip per unit time per unit length along with the interface. Assuming the water lateral diversion length is L_1 , the water lateral flux $Q(x)$ can be calculated by

$$Q(x) \approx r * L_1 < Q_{max} \quad (6)$$

Substituting the equation (6) into equation (5), Equation (7) can be

$$r * L_1 < k_{sat} \tan \beta \int_{\psi_{conc}}^{\psi_{conf}} k_{rw}(\psi) d\psi \quad (7)$$

Diversion length is bounded by

$$L_1 < \frac{k_{sat} \tan \beta \int_{\psi_{conc}}^{\psi_{conf}} k_{rw}(\psi) d\psi}{r} \approx 0.45m \quad (8)$$

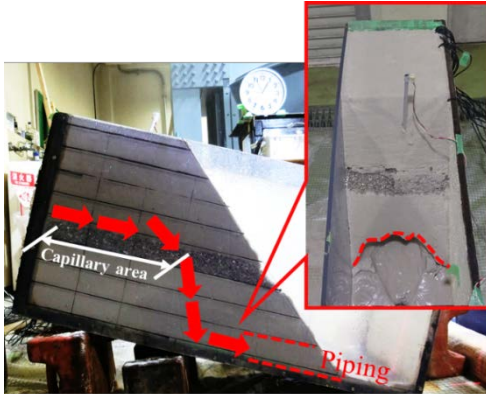


Fig. 17 Accumulated rainfall water infiltrate into the middle part of the slope which cause soil piping

According to the calculation of diversion length by simplified model, length of capillary area (Fig. 17) of this case is smaller than the length of interface. As a result, infiltrating water accumulated above the coarse layer and lateral water flow occurred along with the interface and capillary barrier breakthrough in the middle part of interface which results that water starts to infiltrate into the lower coarse layer at this location. A large amount of rainfall water will pass across the coarse layer and exist in the middle part of the lower fine layer, with rainfall event continued, soil pipes contribute to the quick discharge of rainfall water which results in slope stability.

7. CONCLUSIONS

The following conclusions are drawn from the present study. Different failure modes occurred in under rainfall condition in inclined multi-layer slope and single layer slope.

In the flat group, multi-layer slope was safer since capillary barrier works which prevent the rainwater infiltrate into coarse layer. Failure occurred at the bottom of slope, the toe of slope reach to saturated condition is later while failure time is also late. In inclined group, failure occurred at the same location, Although toe of slope reach to saturated condition is still later, the failure time is earlier.

According to calculate the length of capillary area and compared with experiment pictures, the diversion length is smaller in this case which allows the accumulated rainfall water infiltrate at the middle part of slope, which causes the soil piping and has an influence on the slope stability.

8. REFERENCES

- [1] Mancarella D., Doglioni A. and Simeone V., "On Capillary Barrier Effects and Debris Slide Triggering in Unsaturated Layered Covers," Eng. Geol., vol. 147–148, 2012, pp. 14–27.
- [2] Shackelford S. D., Koohmaraie M. and Savell J. W., "Evaluation of Longissimus Dorsi Muscle pH at Three Hours Post Mortem as a Predictor of Beef Tenderness," vol. 37, 1994, pp. 195–204.
- [3] Garcia E.F., Gallage C.P.K. and Uchimura T., "Function of Permeable Geosynthetics in Unsaturated Embankments Subjected to Rainfall Infiltration," Geosynth. Int., vol. 14, no. 2, 2007, pp. 89–99.
- [4] Ross B., "The Diversion Capacity of Capillary Barriers," Water Resour. Res., vol. 26, no. 10, 1990, pp. 2625–2629.
- [5] Van Genuchten M.T., "A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils1," Soil Sci. Soc. Am. J., vol. 44, no. 5, 1980, pp. 892.
- [6] Kisch M., "The theory of Seepage From Clay-Blanketed Reservoirs," Géotechnique, vol. 9, no. 1, pp. 9–21, 2009.
- [7] Morel-Seytoux H.J. and Nimmo J.R., "Soil Water Retention and Maximum Capillary Drive from Saturation to Oven Dryness," Water Resour. Res., vol. 35, no. 7, 1999, pp. 2031–2041.
- [8] Yang H., Rahardjo H., Leong E.C. and Fredlund, D.G., "A study of Infiltration on Three Sand Capillary Barriers," Can. Geotech. J., vol. 41, no. 4, 2004, pp. 629–643.
- [9] Xie J., Uchimura T., Chen P., Liu J., Xie C. and Shen Q., "A relationship between displacement and tilting angle of the slope surface in shallow landslides," Landslides, vol. 16, no. 6, 2019, pp. 1243–1251.
- [11] Leroux N.R. and Pomeroy J.W., "Modelling capillary hysteresis effects on preferential flow through melting and cold layered snowpacks," Adv. Water Resour., vol. 107, 2017, pp. 250–264.
- [10] Zhan T.L., Li H., Jia G.W., Chen Y.M. and Fredlund D.G., "Physical and numerical study of lateral diversion by three-layer inclined capillary barrier covers under humid climatic," Can. Geotech. J., vol. 1448, no. July, 2014, pp. 1438–1448.
- [12] Tami D., Rahardjo H., Leong E.C. and Fredlund D.G., "Design and laboratory verification of a physical model of sloping capillary barrier," Can. Geotech. J., vol. 41, no. 5, 2004, pp. 814–830.
- [13] Yang H., Rahardjo H., Leong E.C. and Fredlund D.G., "A study of infiltration on three sand capillary barriers," Can. Geotech. J., vol. 41, no. 4, 2004, pp. 629–643.