

DEVELOPMENT OF SOIL FORMATION CORRESPONDING TO CYCLIC STRESS – DERIVED SHEAR STRAIN FUNCTION BASED ON STRATUM INDEX FACTOR

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ABSTRACT : Historically shear strain function and complex shear modulus based on geophysical methods were based on empirical relationship between Seismic Cone Penetration Test (SCPT) and Cone Penetration Test (CPT) method. New geological concept namely CPI model have been developed and applied in mathematical and engineering strains to study shear strain function. Modeling of the mathematical shear strain is done by halving the stratum index factor (SIF). The different types of responses of CPI model are discussed and classified based on numerical methods using degree of settlement and degree of consolidation. The results show that shear strain amplitude or mathematical shear strain is constant by using the modified CPI method where mathematical shear strain indicated is half of the stratum index factor and engineering shear strain indicated is equal to the stratum index factor. The other major area. Last major finding involves the cyclic variation of Poisson Ratio which indicates the anisotropic properties of soft cohesive soil.

Keywords: *Constant plastic index model, Shear strain, Stratum index factor, Volume matrix change, Shear Modulus and Mohr circle.*

1. INTRODUCTION

It is well-known that in 1923 Terzaghi and his consolidation theory brought the dimension-time into Soil Mechanics [1]. The research about a very soft saturated soil ground consolidation due to self-weight. A lot of evaluation of Terzaghi one dimensional theory has been done to compare the finite element numerical prediction value to the measured value. In the case of a three dimensional field, the Biot's theory of consolidation which complex the equilibrium of total stress, continuity of soil mass and strain of soil deformation has been useful (Biot, 1945).

Constant Plastic Index (CPI) model was set up that state: a relationship soil consolidation formation exist volume matrix change of shear stress - strains curve. The curve is derived a shear stress function on amplitude strain (Or, 2001) or mathematical shear - strain (Hudson & Harrison (1977) due to ($\frac{1}{2} \alpha$) half of stratum index factors finding is in the validity of the Mohr circle plot of both stress and strain from SIF derivation in calculating the shear modulus

of the soft consolidated soil whereby the values are all within range, showing again that the modified CPI and SIF goes hand in hand in quick and accurate prediction of the consolidation of soil and also the evaluation of shear modulus especially in soft ground soil hence solving the disadvantages of secondary velocity propagation in that medium. Void ratio function was also expanded to derive more SIF values to extend the moving boundary in order to better study the regional area.

2. OBJECTIVES

The specific objectives of the research are as follows:

- (1) Modification of Constant Plastic Index method on CPI model based on volume matrix change of the consolidation soil.
- (2) Derived shear strain function using Laplace Transform (L T) and cyclic stress formation as stratum index factor for stress-strain function based on CPI Model
- (3) Estimates shear modulus based on integrating Mohr circle.

3.THEORY - MODIFIED CONSTANT PLASTIC INDEX METHOD

The deformation under cyclic stresses as compression of stratum index factors and it is considered a homogeneous soil subjected to sinusoidal shearing stress of angular velocity stress amplitude and strain amplitude. (Teamrat , A.Ghezzehei , and Dani Or (2001)) [3] . The shearing stress applied upon purely elastic soil resulted in proportional and instantaneous strain.

After completing a cyclic load saturation, consolidation and volume matrix change of soil is a steady state case where $F(e) e_o^x = 1 + e_o$ (Poulos (1981)) (1)

The equation (S_{sul}) derived will become a mathematical equation as

$$e_o^x = \frac{\Delta e}{S_{sul}} H \quad (2)$$

Where H is thickness of soil layer
 Δe is a (final void ratio –initial void ratio)
 e_o is initial void ratio

S_{sul} is Ultimate Consolidation Settlement

The equation 2 can be expressed as log10.

$$\log_{e_o} e_o^x = \log_{e_o} \left[\frac{\Delta e H}{S_{sul}} \right] \quad (3)$$

$$\log_{e_o} e_o^x = \log_{e_o} (\Delta e H) - \log_{e_o} S_{sul}$$

$$X = \frac{\ln \left(\frac{\Delta e H}{S_{sul}} \right)}{\ln e_o} \quad (4)$$

Where, x is Constant Plastic Index (CPI)
 $F(e)$ is Void ratio Function of the potential yields function Elastic soil (Hardin Bankford, 1989) and the equation is follows Hooke Law for a number of clay $F(e) = e_o^x$ (5)

Where $F(e)$ is void ratio function

X is constant plastic index (CPI)

e_o is initial void ratio

in layer hypothesis in Fig. 1 is considered with an initially uniform void ratio e_1 under its own weight, and e_2 is the base of layer which suddenly increased

In situation where single drainage is only allowed (Kuantsai 1979)[1] at one of the two boundaries of the stratum, the progress of consolidation will be retarded. In the theory of a linear soil model and the thin layer the actual location of this boundary, whether it is in the surface or the base, does not effected the

the stress, then remained constant. per surface of this layer is allowed to drain freely , hence its void ratio will immediately reach the new equilibrium state, e_2 . The base of this layer is assumed to be impervious and thus, no drainage can occur across it.

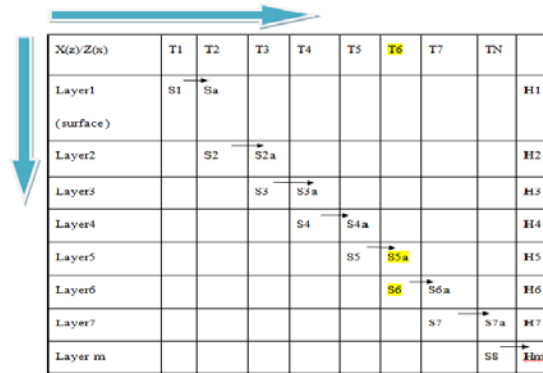


Fig. 1 Surface and base of the layer .

In the Langrangian formulation, the compaction within two layers is complicated because of the presence of the fixed boundary. The Constant plastic index (CPI) mechanism of compacted clay between two layers is considered to be integrate Stratum Index Factors (α) , A characteristic of compacted soil between surface (s) and base (b) of layers is formulated as

$$\alpha = \ln \frac{e_s^{x1}}{e_b^{x2}} \quad (6)$$

Furthermore, to obtain the evaluation formula for the expansion characteristic of compaction, it is proposed that the development of CPI (x) model and Stratum Index Factor (SIF) model be combined to get the estimate of the CPI (x) characteristic of clay.

4. THEORY - DERIVED SHEAR STRAIN BASED ON STRATUM INDEX FACTOR

strain rate of the stratum. This is because identical void ratio changes occur at both boundaries. In the present theory, however, the void ratio changes in both boundaries will be different. Consequently, a difference in the strain rate is expected for different locations of the drainage boundary. This will be

confirmed in this section by the solutions developed for each case.

The inverse transformation according to the inversion theorem is evaluated in (Lee Kuantsai 1979, pg166) and which it is found that:

$$\beta_n \cos \beta_n + \frac{\alpha}{2} \sin \beta_n = 0 \quad \text{shear strain function and}$$

$$\beta_n \cos \beta_n - \frac{\alpha}{2} \sin \beta_n = 0 \quad \text{shear tensile function (7)}$$

Surface drain The boundary conditions in this case are:

$$q(1, Tv) = \Delta F(e) \quad (8)$$

$$\frac{\partial q}{\partial n}(0, Tv) = \alpha q(0, t) \quad (9)$$

Using the Laplace transform it is found (Lee Kuantsai , 1979) that:

$$\bar{q}(n, p) = A_1 \exp(m_1 n) + A_2 \exp(m_2 n), \quad \begin{matrix} m_1 = \\ m_2 = \end{matrix}$$

$$\frac{\alpha}{2} \pm \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \quad (10)$$

with,

$$A_1 = \frac{B \Delta F(e)}{p[\exp(m_2) - B \exp(m_1)]}$$

$$A_2 = \frac{a \exp(m_2) - b}{p[\exp(m_2) - \exp(m_1)]} \quad \text{and} \quad B = \frac{m_2 - \alpha}{m_1 - \alpha}$$

The Laplace transform of the local degree of consolidation is given by:

$$\bar{f}(n, p) = \frac{\bar{q}(n, p)}{q(n, \infty)}$$

After some manipulation, this is found to be:

$$\bar{f}(n, p) =$$

$$\frac{\exp\left[-\frac{\alpha}{2}(n-1)\right] \left\{ \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh\left[n \sqrt{\left(\frac{\alpha^2}{4} + p\right)}\right] + \frac{\alpha}{2} \sinh\left[n \sqrt{\left(\frac{\alpha^2}{4} + p\right)}\right] \right\}}{p \left\{ \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh\left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)}\right] + \frac{\alpha}{2} \sinh\left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)}\right] \right\}}$$

(11)

The inverse transformation according to the inversion theorem is evaluated in (Lee Kuantsai 1979, pg166) and Fig. 2 from which it is found that:

$$f(n, Tv) = 1 -$$

$$2 \sum_n \frac{(-1)^n [\beta_n \cos(\beta_n n) + \frac{\alpha}{2} \sin(\beta_n n)]}{\sqrt{\left(\beta_n^2 + \frac{\alpha^2}{4}\right)} [\beta_n + \frac{\alpha^2}{4\beta_n} + \frac{\alpha}{2\beta_n}]} \exp\left[-\left(\beta_n^2 + \frac{\alpha^2}{4}\right) Tv - \frac{\alpha}{2}(n-1)\right]$$

where the β_n s are the zeroes of

$$\beta_n \cos \beta_n + \frac{\alpha}{2} \sin \beta_n = 0 \quad \text{as shear strain and shear rate function}$$

The expression for the void ratio change q can be obtained by:

$$q(n, Tv) = \Delta F(e) \exp[\alpha(n-1)] f(n, Tv)$$

The degree of settlement is found to be:

$$S(Tv) = 1 -$$

$$\frac{2\alpha}{1 - \exp(-\alpha)} \sum_n \frac{\exp\left[-\left(\beta_n^2 + \frac{\alpha^2}{4}\right) Tv\right]}{\left(\beta_n^2 + \frac{\alpha^2}{4}\right) \left(1 + \frac{\alpha^2}{4\beta_n^2} + \frac{\alpha}{2\beta_n}\right)}$$

The small time approximation to this can be obtained by considering the behaviour of the Laplace transform, which is:

$$S(p) = \frac{\alpha}{p[1 - \exp(-\alpha)]} \frac{1}{\sqrt{\left(\frac{\alpha^2}{4} + p\right)} \coth\left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)} + \frac{\alpha}{2}\right]}$$

As $p \rightarrow \infty$, $\coth \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \rightarrow 1$

It follows that :

$$s(p) = \frac{\alpha}{1-\exp(-\alpha)} \frac{1}{p \left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)} + \frac{\alpha}{2} \right]} \quad p \rightarrow \infty$$

Since,

$$L^{-1} \left\{ \frac{1}{p \left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)} + \frac{\alpha}{2} \right]} \right\} = L^{-1} \left\{ \frac{\sqrt{\left(\frac{\alpha^2}{4} + p\right)}}{p^2} \right\} - \frac{\alpha}{2} T v$$

In large time an approximate solution can again be obtained by taking the first few terms of the series solution, eq.12 . A comparison of the small and large time approximation (up to the third term in the series) with the exact solution is shown in a typical value of $\alpha=1$. The agreement is seen to be very good.

(b) Based drain

The boundary conditions for this case are:

$$q(0, Tv) = \Delta F(e) \exp(-\alpha)$$

$$\frac{\partial q}{\partial n} (1, Tv) = \alpha q(1, Tv)$$

Using the Laplace transform it is found that:

$$\bar{q}(n, p) = \frac{\Delta F(e) \exp\left(\frac{\alpha}{2} n - \alpha\right)}{p} \left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh \left[(1-n) \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \right] - \frac{\alpha}{2} \sinh \left[(1-n) \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \right] \right]$$

It follows from eq.11 that,

$$L^{-1} \left\{ \frac{\sqrt{\left(\frac{\alpha^2}{4} + p\right)}}{p^2} \right\} \simeq 2 \sqrt{\left(\frac{Tv}{\pi}\right)} + \frac{\alpha^2}{2\sqrt{\pi}} \sqrt{(Tv)^3}$$

Which is,

$$S(Tv) = \frac{\alpha}{1-\exp(-\alpha)} \left[2 \sqrt{\left(\frac{Tv}{\pi}\right)} - \frac{\alpha}{2} Tv + \frac{\alpha^2}{6\sqrt{\pi}} \sqrt{Tv^3} \right] \quad T \rightarrow 0 \quad (12)$$

$$p) \left] \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh \sqrt{\left(\frac{\alpha^2}{4} + p\right)} - \frac{\alpha}{2} \sinh \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \right]$$

and,

$$\bar{f}(n, p) =$$

$$\frac{\exp\left(-\frac{\alpha}{2} n\right)}{p} \frac{\sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh \left[(1-n) \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \right] - \frac{\alpha}{2} \sinh \left[(1-n) \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \right]}{\sqrt{\left(\frac{\alpha^2}{4} + p\right)} \cosh \sqrt{\left(\frac{\alpha^2}{4} + p\right)} - \frac{\alpha}{2} \sinh \sqrt{\left(\frac{\alpha^2}{4} + p\right)}}$$

The inverse transform of this is also evaluated in Lee Kuantsai page 169 and 59 and this chapter, from which it is found that:

$$f(n, Tv) = 1 -$$

$$2 \sum_n \frac{(-1)^n \left[\alpha_n \cos \alpha_n (1-n) - \frac{\alpha}{2} \sin \alpha_n (1-n) \right]}{\left(\alpha_n + \frac{\alpha^2}{4} \right) \left(\alpha_n^2 + \frac{\alpha^2}{4\alpha_n} - \frac{\alpha}{2\alpha_n} \right)} \exp \left[- \left(\alpha_n^2 + \frac{\alpha^2}{4} \right) Tv - \frac{\alpha}{2} n \right]$$

and the degree of settlement is:

$$S(Tv) = 1 -$$

$$\frac{2\alpha}{\exp(\alpha)-1} \sum \frac{\exp\left[-\left(\alpha_n^2 + \frac{\alpha^2}{4}\right)Tv\right]}{\left(\alpha_n^2 + \frac{\alpha^2}{4}\right)\left(1 + \frac{\alpha^2}{4\alpha_n^2} - \frac{\alpha}{2\alpha_n^2}\right)}$$

where the α_n are the zeroes of

$$\alpha_n \cos \alpha_n - \frac{\alpha}{2} \sin \alpha_n = 0 \quad (13)$$

as a shear strain and shear rate function

Plasticity soil . The small time approximation to the degree of settlement is obtained by Lee Kuantsai (1979) and considering its Laplace transform:

$$\bar{S}(p) = \frac{\alpha}{p[\exp(\alpha)-1]} \frac{1}{\sqrt{\left(\frac{\alpha^2}{4} + p\right)} \coth \sqrt{\left(\frac{\alpha^2}{4} + p\right)} - \frac{\alpha}{2}}$$

equation 14

$$\text{As } p \rightarrow \infty, \quad \coth \sqrt{\left(\frac{\alpha^2}{4} + p\right)} \rightarrow 1$$

it follows that:

$$S(p) = \frac{\alpha}{\exp(\alpha)-1} \frac{1}{p \left[\sqrt{\left(\frac{\alpha^2}{4} + p\right)} - \frac{\alpha}{2} \right]}$$

The inverse transform of this is, approximately in Fig.

13

$$S(T) = \frac{\alpha}{\exp(\alpha)-1} \left[2\sqrt{\left(\frac{Tv}{\pi}\right)} + \frac{\alpha}{2}Tv - \frac{\alpha^2}{6} \frac{1}{\sqrt{\pi}} \sqrt{Tv^3} \right] \quad (14)$$

It is worth noting that by adding this to the expression for the surface drain case:

$$S_{base} + S_{surface} = \frac{\alpha}{\exp(\alpha)-1} [2(1 + \exp(\alpha))\sqrt{\left(\frac{Tv}{\pi}\right)} - \frac{\alpha}{2}(\exp(\alpha) - 1)Tv + \frac{\alpha^2}{6}(1 + \exp(\alpha))\frac{1}{\sqrt{\pi}}] \quad (15)$$

The inverse transform of this is also evaluated in Lee Kuantsai page 231 and in this chapter, from which it is found that:

$$f(n, Tv) = 1 -$$

$$2 \sum_n \frac{(-1)^n \left[\alpha_n \cos \alpha_n (1-n) - \frac{\alpha}{2} \sin \alpha_n (1-n) \right]}{\left(\alpha_n^2 + \frac{\alpha^2}{4} \right) \left(\alpha_n^2 + \frac{\alpha^2}{4\alpha_n^2} - \frac{\alpha}{2\alpha_n^2} \right)} \exp \left[- \left(\alpha_n^2 + \frac{\alpha^2}{4} \right) Tv - \frac{\alpha}{2} n \right]$$

Degree of settlement is:

$$S(Tv) = 1 - \frac{2\alpha}{\exp(\alpha)-1} \sum \frac{\exp\left[-\left(\alpha_n^2 + \frac{\alpha^2}{4}\right)Tv\right]}{\left(\alpha_n^2 + \frac{\alpha^2}{4}\right)\left(1 + \frac{\alpha^2}{4\alpha_n^2} - \frac{\alpha}{2\alpha_n^2}\right)} \quad (16)$$

5. RESULT

CPI model is a single drainage soil compaction model by externally applied stress Consolidation of soil results in the modification of constant plastic index because of the influences of such changes on the degree of consolidation and degree of settlement process. Soil deformation is time-dependent on the derived stratum index factor of soil that affects both the shear strain and shear stress function. Both parameters are presented in the Mohr circle curve and it is given in space and time to accurately predict the shear modulus of the homogeneous soft soil.

Detailed study on the consolidated soil shows that the CPI model has been integrated with new parameters, mainly shear stress function and shear strain function. The effect of shear stress on the shrinkage and creep behavior of soil is defined as shear stress function (Horn & Baumgartl, 2002). The shear strain function occurs in a two phase system (solid and liquid). The two systems can be described shortly as plastic and elasticity of soil. Strain amplitude for a single base is 0.9567 and hence is a positive SIF of 1.913 for elastic soil and tensile is -0.9567, where the negative SIF amounting to -1.913 which is described as plastic conditions. The time axis is scaled by the

frequency (angular velocity) of loading and is expressed in units of degrees [3] . Peat earth is a type of plastic soil and it was observed being heaved above the bed of Sg. Talam forming a small island (Shear strain amplitude was -0.14 and stratum index factor was - 0.1380, which showed tensile forces as per Fig. 4.

The progress of the degree of settlement with the square root of time factors for both surface and base drain cases . The differential of pore water pressure and void ratio change absent and the degree of consolidation will move to lower consolidation . In the surface drain the trend is similar to the double drainage case in that faster consolidation achieved with larger SIF . However the opposite is observed in the base drain case where larger SIF will produce a slower consolidation , and it can be seen that the consolidation with base drain is always slower than the thin layer , while the surface drain will always be faster . It can be seen that the consolidation with stratum coefficient (Kuantsai 1977) is always slower than the SIF[1] . A comparison of the small and large time approximation (up to the third term in the series) with the exact Fig. 1 Single drainage for surface shear strain

Fig. 1 Single drainage for base shear strain function . Single Surface - Case 2— $t=Constant$, C_v and C_F = various . Single surface CPI $S(T_v)$ increased due to Lee Kuantsai method.

6 CONCLUSION

Therefore the contributions of the CPI model are in terms of the estimation technique to evaluate the amplitude of shear strain and shear stress function. By using the modified CPI model, shear modulus can be predicted where Seismic Cone Penetration Test (SCPT) cannot propagate shear waves. Conclusion has been made that half of stratum index factor equals to amplitude of shear strain. The major findings as in the summary section of this chapter shows that this thesis contributes much more than what it is stated here. This thesis gives an overall picture of soil mechanics determination in terms of volume matrix change, constant plastic index, void ratio function, stratum index factor, stress strain regimes and shear modulus to study soil consolidation, soil settlement and cyclic stresses.

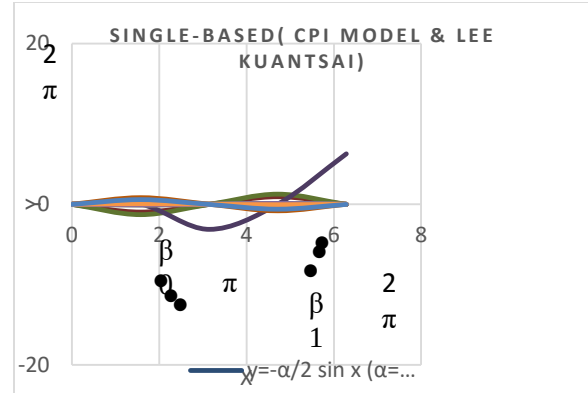


Fig. 2 Shear strain function for single surface CPI model .

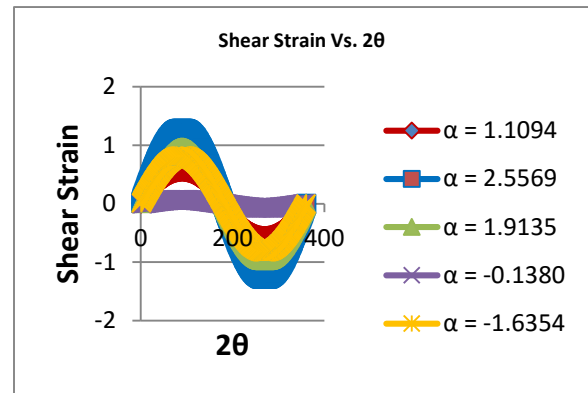


Fig. 3 Shear strain versus 2θ

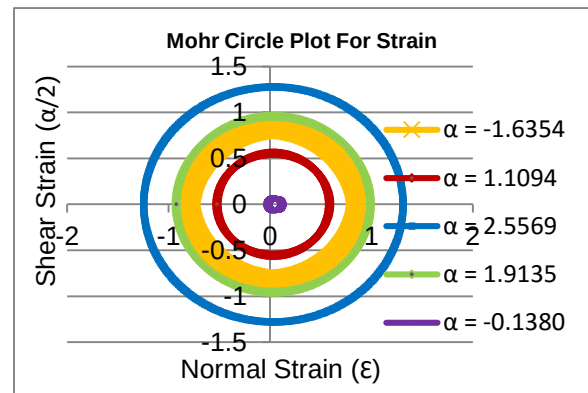


Fig. 4 Shear strain versus normal strain

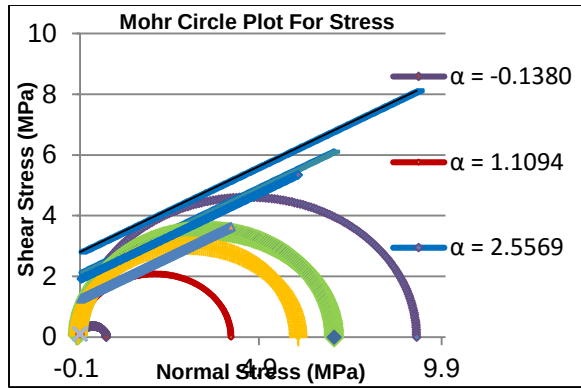


Fig. 5 Shear stress versus normal stress

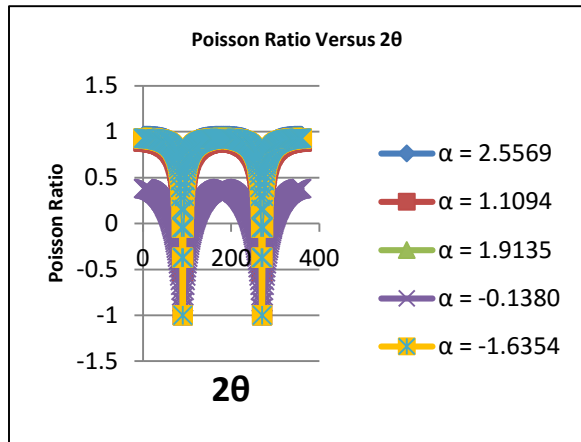


Fig. 6 Poisson ratio versus rotation 2θ

SIF (α)	Shear Strain ($\alpha/2$)	Shear Stress, τ (MPa)	Shear Modulus, G (MPa)
2.5569	1.2785	4.609	3.6051
1.9135	0.9568	3.5	3.6582
1.1094 (model)	0.5547	2.08	3.7498
-1.6354	-0.8177	3.01	-3.6811
-0.138	-0.069	0.362	-5.2464

Fig. 7 Shear modulus table values derived from SIF and shear stress function .

As a result, the greatest contribution is in the long term application which is twofold: firstly to bring economic benefit by improving the consolidation rate of soft cohesive soil and to quickly determine the stress strain regime on a regional scale in the most practically accurate way possible, and secondly to provide a platform for further subsequent research.

7. REFERENCES

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