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RELATIONSHIP BETWEEN LIQUEFACTION STRENGTH OF SANDY SOIL WITH FINE FRACTION

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ABSTRACT: The author's research aims to clarify the relationship between fine fraction content, which represents the soil's skeletal structure, and liquefaction strength. In previous papers, we reported on the behavior of using Toyoura sand (D_{50}=0.16mm) as the coarse grain fraction and various fine grain fractions (silt and clay) using the fine fraction void ratio as a parameter. Good relationships between the fine fraction void ratio and liquefaction strength were observed. In this paper, we added Tohoku silica sand (D_{50}=0.67mm), which has larger coarse grains than Toyoura sand. As a result, we found it impossible to compare soils with different maximum and minimum void ratios for unity estimation. The results of this study showed that it is important to revise the index that describes the state of sand based on its grain size and fine fraction content. It also became clear that, for the sand used in this study, the skeleton void ratio and skeleton relative density are well correlated with the liquefaction strength.

Keywords: Liquefaction, Sand, Fine fraction, Skeleton structure

1. INTRODUCTION

Liquefaction caused major damage to civil engineering structures, as evidenced by the 1964 Niigata Earthquake and the 1983 Nihonkai-Chubu Earthquake. However, the liquefaction caused by these earthquakes occurred in pure sandy ground with uniform grain size [1].

Consequently, many researchers have focused their attention on soils containing loosely deposited saturated sand rather than coarse sand containing gravel or sand with a high content of fine fractions. In particular, sand containing fine particles has not been thoroughly researched due to the difficulty of liquefying fine particles.

However, the 1987 Chiba Prefecture East Coast Earthquake, the 1989 Loma Prieta Earthquake, the 1995 Hyogo Prefecture Southern Earthquake, the 2000 Tottori Prefecture Western Earthquake, and the 2011 Tohoku Pacific Coast Earthquake revealed that reclaimed ground containing many fine grains, previously thought difficult to liquefy, did liquefy. Severe damage also occurred to many civil engineering structures and residential houses. The reclaimed land containing decomposed granite that liquefied during the Hyogo earthquake contained various grain sizes, including gravel [2].

In this context, many researchers in Japan have conducted studies to clarify the liquefaction mechanism and the influence of soil particle size on the liquefaction strength of soils containing fine particles, such as silt and clay, as well as sandy soils containing large particles, such as gravel. In particular, Koseki et al. [3] reported on the liquefaction strength of sand containing fine fractions. They showed that the higher the plasticity index, the higher the liquefaction strength; conversely, in low-plasticity samples, liquefaction strength decreases as the proportion of fine particles increases. Since then, the plasticity index has been used to predict liquefaction. On the other hand, Yajima et al. [4] reported experimental data that showed that sandy ground containing a certain amount of fine fraction is prone to liquefaction. Nabeshima's [5] study examined fine-grained soil, divided it into silt and clay fractions, and proposed a method that utilizes the framework void ratio. This method considers the clay fraction as voids. Sato [6] researched the plasticity of fine particles and reported that an increase in non-plastic fine particles results in a dramatic decrease in liquefaction strength, even at fine particle contents of around 50%. This means that there is no unified evaluation of the effect of fine particle content on the liquefaction resistance of sandy soil. In recent years, Porcino [7] and Stefania et al. [8] conducted various studies on the fine fraction content and its effect on liquefaction strength. Therefore, this study was conducted to understand the characteristics of liquefaction strength, for which specimens were prepared using the same method with varying coarse- and fine-grained component contents. Liquefaction tests were performed on sand specimens containing fine-grained soils with different plasticity indexes mixed in the range Fc = 0-40% (Fc: fine fraction content), in order to unify the liquefaction strength of sand containing fine-grained soils.

In previous studies, the authors have reported experimental results on the relationship between fines content and liquefaction strength, as well as the relationship between fines properties and liquefaction strength [11-13].

In this paper, the effects of particle size were examined by comparing coarse-grained soils of different particle sizes with those used previously.

As a result, it was found that by using an index called skeletal relative density, it is possible to uniformly evaluate the relationship between liquefaction strength and fine particle content in a range where fine particle content is not too large.

2. RESEARCH SIGNIFICANCE

The content of fine-grained material is widely recognized as a key factor in determining the liquefaction strength of sandy soils. While many previous studies have investigated the effects of varying the type of fine-grained material, very few have examined the effects of varying the type of coarse-grained material. This study conducted experiments by varying the types and contents of both the coarse and fine-grained fractions while maintaining the same sample preparation method. The results showed that using 'skeletal relative density' as a parameter provides a consistent evaluation of liquefaction resistance.

3. EXPERIMENTAL OVERVIEW

3.1 Sample Soil

The samples used were Toyoura sand (D_{50}=0.161 mm, fine sand) as the coarse-grained soil and Tohoku Silica Sand No. 4 (D_{50}=0.666 mm, medium sand), which has a larger grain size than Toyoura sand. Hereafter, it is called Tohoku sand. Fine-grained sand (sandy soil) was meticulously prepared to ensure accurate mixing of fine and coarse grains. Kaolin clay (D_{50}=0.003 mm, clay), Fujinomori silt (D_{50}= 0.009 mm, silt), and Kasaoka clay (D_{50}=0.006 mm, silt) were used as fine-grained soils to examine the effect of differences in plasticity index. The completed specimen was considered uniform. The typical properties and grain size distribution of each sample are shown in Tables 1 - 3, and Fig. 1. These samples were mixed to prepare four types of soil: Toyoura-kaolin mixed soil, Toyoura-Fujinomori mixed soil, Toyoura-Kasaoka mixed soil, and Tohoku-Kasaoka mixed soil. The mixing ratio was the percentage of the dry mass of the fine particles to the total dry mass, and the fine particle content Fc of the mixed soil was 0 to 40%.

Table 1 Typical properties of coarse-grained soil

Coarse-grained soil	Soil particle ρ_s (Mg/m^3)	Mean grain size D_{50} (mm)	Maximum void ratio e_{max} (-)	Minimum void ratio e_{min} (-)
Toyoura sand	2.64	0.16	0.97	0.62
Tohoku sand (#4)	2.66	0.67	0.76	0.52

Table 2 Typical properties of fine-grained soil

Fine-grained soil	Soil particle density ρ_s (Mg/m^3)	Mean grain size D_{50} (mm)	Clay fraction content Cc (%)	Fine fraction content Fc (%)	Plasticity index IP (-)
Kaolin clay	2.71	0.0030	64.0	100.0	13.7
Fujinomori silt	2.54	0.0070	37.0	92.0	20.4
Kasaoka clay	2.71	0.0027	46.0	100.0	30.8

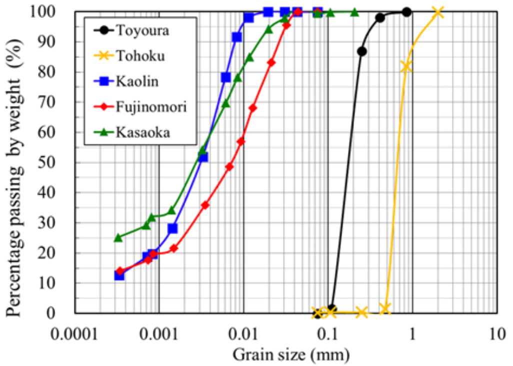


Fig. 1 Grain size accumulation curves of the samples

Table 3 Typical properties of mixed soil

Sample	Fine fraction content Fc (%)	Soil particle density ρ_s (g/cm^3)	Plasitisity index Ip(-)
Toyoura + Kaolin	35	2.66	5.4
	40	2.67	8.0
	60	2.68	10.3
Toyoura + Fujinomori	30	2.63	6.6
	40	2.63	12.6
Toyoura + Kasaoka	30	2.66	8.6
	40	2.67	11.4
	60	2.68	15.6
Tohoku + Kasaoka	30	2.68	5.0
	40	2.68	11.0

3.2 Test Method and Condition

This study used a cyclic undrained triaxial apparatus to determine each sample's liquefaction strength (see Fig. 2). The test specimens, cylindrical with a diameter of 50 mm and a height of 100 mm, were prepared using the following method. The test specimens were prepared by dividing the dry coarse-grained and dry fine-grained fractions into 10 equal mass portions, thoroughly mixing them until homogeneous, and compacting each layer to the specified height (density) using a rammer. After that, water was passed through the sample soil to confirm its saturation state, and then the soil was consolidated at an effective confining pressure of σ_c' = 100 kPa.

The liquefaction strength of each sample soil was measured by repeatedly loading the soil with a sine wave of a frequency of 0.1 Hz at a specified repeated stress ratio R (= $\sigma_d/2\sigma_c'$). Relative density is used as a density control parameter for sand test specimens. Still, since the Japanese Industrial Standard (JIS) does not apply when the fine particle content is 5% or more, the authors decided to use the same value as the coarse particle void ratio. Table 4 shows the density control for each soil sample tested.

Table 4 Density of each soil sample

Sample	Relative density Dr (%)	Void ratio e (-)	Condition -
Toyoura sand	80	0.684	Dence
	60	0.757	Medium
	40	0.858	Loose
Tohoku sand	60	0.617	Medium
	40	0.666	Loose
Toyoura + Kaolin	80	0.684	Dence
	60	0.757	Medium
	40	0.828	Loose
Toyoura + Fujinomori	60	0.757	Medium
Toyoura + Kasaoka	60	0.757	Medium
Tohoku + Kasaoka	60	0.617	Medium

Note: Since the JIS standard stipulates that the fine content should be less than 5%, the authors set the void ratio to the same value as the coarse material for mixture-soils fine particles containing 5% or more.

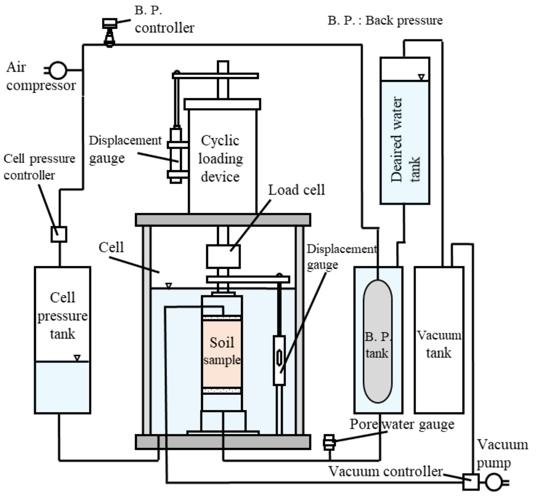


Fig. 2 Cyclic undrained triaxial test apparatus

4. EXPERIMENTAL OVERVIEW

4.1 Liquefaction Strength Curves

4.1.1 Coarse-grained soil

Fig. 3 shows the liquefaction strength curves of the specimens containing only coarse-grained soil (Toyoura sand and Tohoku sand). The liquefaction strength increases with increasing density in both cases. The liquefaction strength of Tohoku sand is also more significant than that of Toyoura sand. The reason is the difference in the two's average grain size (D_{50}).

To examine the validity of these experimental results conducted using coarse-grained soil, Fig. 4 compares Ishihara’s liquefaction strength R_{20} with each sample soil's liquefaction strength (R_{20}) [14]. Ishihara proposed the relationship between the liquefaction strength, R_{20}, obtained from cyclic undrained triaxial tests and relative density. The results of this experiment were also close to those suggested by Ishihara.

4.1.2 Fine-grained soil

Figs. 5 -8 present the liquefaction strength curves in repeated triaxial compression tests, a significant finding when various fine particles were mixed with the aforementioned coarse-grained soil in a specified ratio.

The liquefaction strength curves from cyclic undrained triaxial tests conducted on Toyoura-

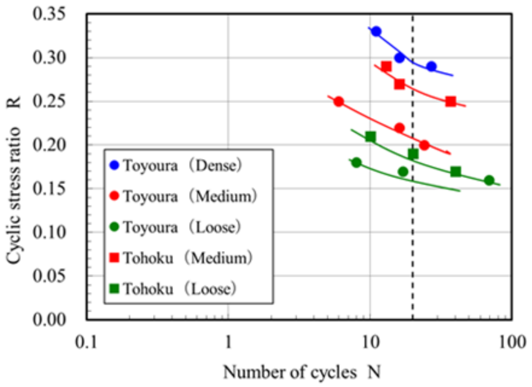


Fig.3 The liquefaction strength curves of the coarse-grained soil

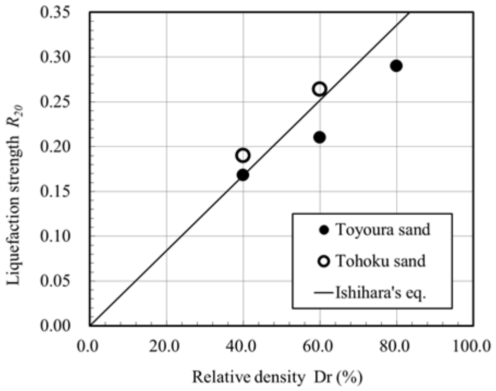


Fig. 4 The relationship between liquefaction strength and relative density of the coarse-grained soil

kaolin mixed soil are shown in Figs. 5 and 6. Liquefaction strength curves of the mixed soil with densities (Fc) of 5%, 10%, 15%, 35%, and 40% were tested. Here, Fc indicates the fine fraction content and is defined as the dry mass of fine soil (clay and silt) relative to the dry mass of coarse soil (sand).

As with Toyoura sand, the liquefaction strength curves shift upward with increasing density, and resistance to liquefaction increases, even at Fc=5% and 10%. Still, the increase in the fine particle content tends to be smaller. In particular, there was almost no increase in liquefaction resistance with increasing density in samples with Fc=15%, 35%, and 40%. In addition, the liquefaction strength decreased up to about Fc=20%. Still, from Fc=25% onwards, the liquefaction strength gradually increased, providing a significant insight into the behavior of fine-grained soil in triaxial compression tests.

Fig. 7 shows the liquefaction strength curves from a cyclic undrained triaxial test conducted using Toyoura and Fujinomori mixed soil. For Toyoura and Fujinomori mixed soil, all samples with different mixture ratios were adjusted to a medium density for testing, so the results were compiled into a single graph. For Fc up to 20%, the

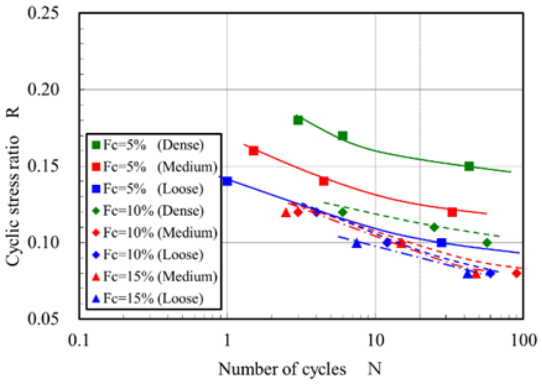


Fig. 5 The relationship between cyclic stress ratio and Number of cycles of Mixture soil of Toyoura sand and Kaolin clay (Fc =5% to 15%)

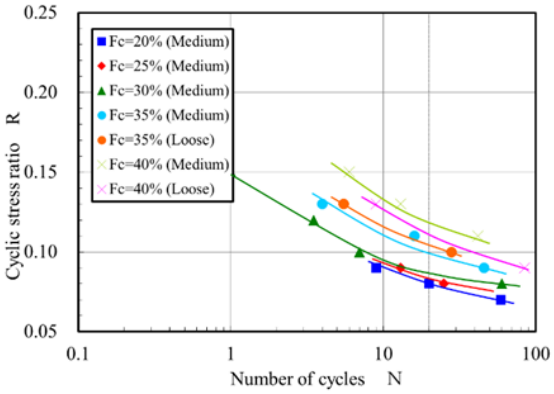


Fig. 6 The relationship between cyclic stress ratio and Number of cycles of Mixture soil of Toyoura sand and Kaolin clay (Fc =20% to 40%)

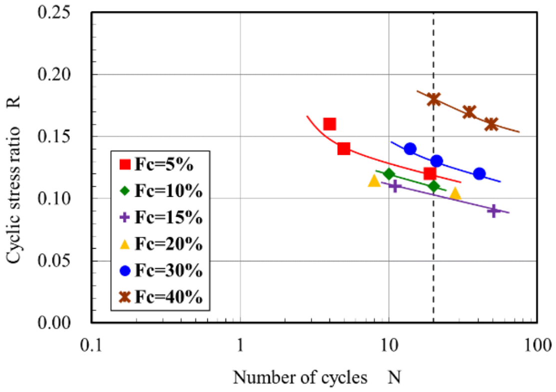


Fig. 7 The relationship between cyclic stress ratio and Number of cycles of Mixture soil of Toyoura sand and Fujinomori clay (Medium)

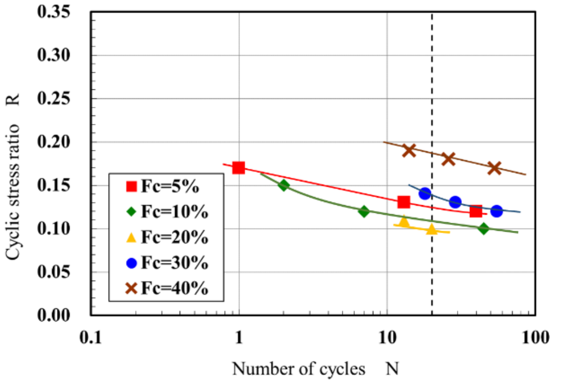


Fig. 8 The relationship between cyclic stress ratio and Number of cycles of Mixture soil of Toyoura sand and Kasaoka clay (Medium)

curve is at the bottom, indicating low resistance to liquefaction, with significant implications for soil engineering. Beyond that, the curve peaks, indicating an increase in liquefaction strength, further underscoring the practical relevance of our research.

Fig. 8 shows the liquefaction strength curve when a cyclic undrained triaxial test using Toyoura and Kasaoka mixed soil. The curve looks the same as the Toyoura and Fujinomori mixed soil.

4.2 Relationship between Liquefaction Strength and Fine Fraction Contents

Figs. 9-11 shows the relationship between liquefaction strength R_{20} and fine fraction content Fc. Fig. 9 focuses on the density differences of samples, Fig. 10 focuses on the differences in types of fine-grained soil, and Fig. 11 focuses on the differences in types of sand. In all mixed soils, the liquefaction strength R_{20} decreases with increasing fine fraction content, but begins to increase at around Fc = 20%. This tendency aligns with previous studies. Fig. 9 shows that the liquefaction strength increases with increasing density in the sand-only sample (Fc = 0%). However, as the fine fraction content (Fc) increases, the increase in liquefaction strength R_{20} due to density increase becomes smaller, and at Fc = 15%, Loose and Medium show almost the same liquefaction strength R_{20}. From this, it can be seen that the effect of density differences on liquefaction strength is significant in areas with low fine content but small in areas with high fine content. Fig. 10 shows no difference in liquefaction strength depending on the type of fine soil when the fine content is around Fc = 15%. However, from Fc = 20% onwards, differences in liquefaction strength gradually appear depending on the type of fine soil. Even with the same density, as the fine content increases, there is a tendency for liquefaction strength to change depending on the type of fine fractions. Fig. 11 shows that, before Fc = 5%, liquefaction strength varies with the type of coarse fractions. However, from Fc = 10% onwards, the liquefaction strength becomes almost the same, and it was found that the liquefaction strength is nearly the same regardless of the type of sand.

4.3 Skeleton Void Ratio and Plasticity Index of Sand-Mixed Soil

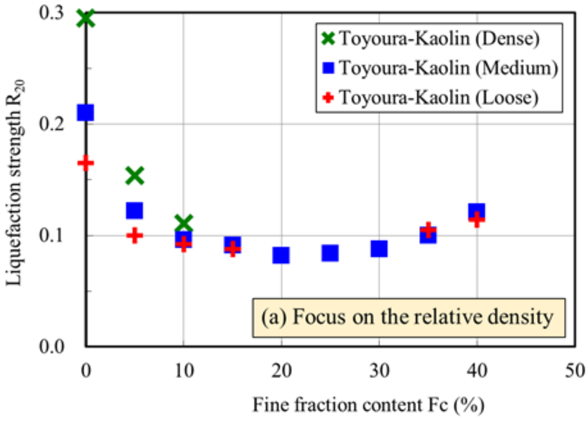


Fig. 9 The relationship between fine fraction content and Liquefaction strength (focus on the relative density)

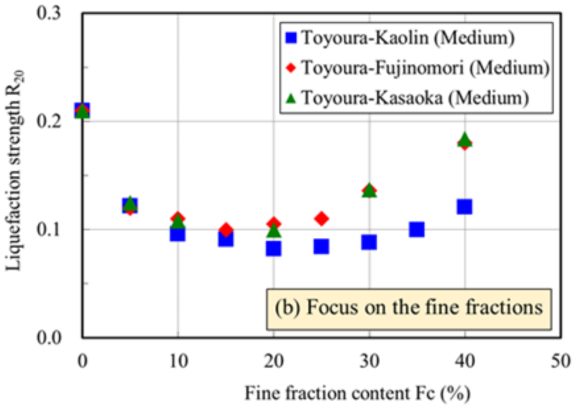


Fig. 10 The relationship between fine fraction content and Liquefaction strength (focus on the type of fine fraction)

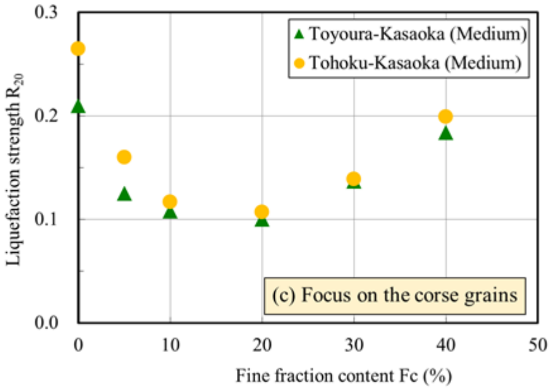


Fig. 11 The relationship between fine fraction content and Liquefaction strength (focus on the coarse grains)

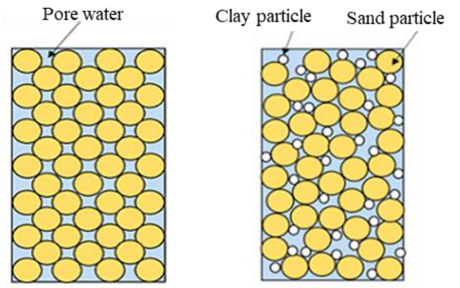
This paper is thorough, examining the relationship between the fine content Fc and liquefaction strength R_{20} in Fig. 12. It appears that density is related in areas with low fine content, whereas parameters that represent clay properties are related in areas with high fine content. Therefore, we decided to examine the relationship between the skeleton void ratio e_s [6]

and liquefaction strength R_{20} in areas with low fine content and the relationship between the plasticity index and liquefaction strength of mixed soil in areas with high fine content. The skeleton void ratio e_s is expressed by Eq. (1) and is the void ratio when all fine particles are considered voids and represent the density of sand only. Fig. 12 shows a schematic diagram of the skeleton structure of mixed soil. When the skeleton void ratio value is between the minimum void ratio e_{min} of sand in Fig. 12 (a) and the maximum void ratio e_{max} of sand in Fig. 12 (c), the fine particles enter the voids of the sand in Fig. 12 (b). Sand particles interlock to form a sand skeleton.

Skeleton void ratio e_s

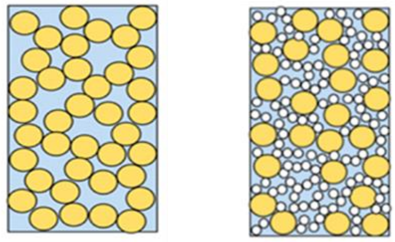
$$e_s = \frac{V_v + V_{s(silt)} + V_{s(clay)}}{V_{s(sand)}} \qquad (1)$$

where, V_v : Volume of void
V_s : Volume of soil particles



(a)Minimum void ratio of sand particle e_{min}

(b) Sand particle's skeleton $e_{min} < e_s < e_{max}$



(c) Maximum void ratio of sand particle e_{max}

(d) Clay particle's skeleton $e_{min} < e_s$

Fig. 12 Schematic diagram of the skeleton structures of sand and mixed soils

4.4 Relationship between Skeleton Void Ratio and Liquefaction Strength

Fig. 13 shows the relationship between the skeleton void ratio es and the liquefaction strength R_{20} for each mixed soil. The red trend line represents the Toyoura sand-mixed soil, and the yellow trend line represents the Tohoku sand-mixed soil. The red dashed line range indicates the range of the sand skeleton of Toyoura sand, and the yellow dashed line range indicates the range of the sand skeleton of Tohoku sand. In the range of more petite than the maximum void ratio, e_{max} of sand alone, the liquefaction strength decreases with an increase in the skeleton void ratio. The skeleton void ratio represents the density of sand, with fine particles treated as voids. The liquefaction strength also decreased because the sand's density decreased after mixing fine fractions. In addition, the fine particles enter the voids in the sand, and the skeleton is formed only by the sand, so the fine particles do not contribute to the liquefaction strength. However, since the ranges of e_{min} and e_{max} for sand differ by type, they are shown in two curves. Therefore, since it is considered difficult to evaluate the difference in the sand with different particle sizes only by the skeleton void ratio, it is considered that the skeleton relative density, which is an index showing where the skeleton void ratio is located between the e_{max} and e_{min} of the sand, is appropriate.

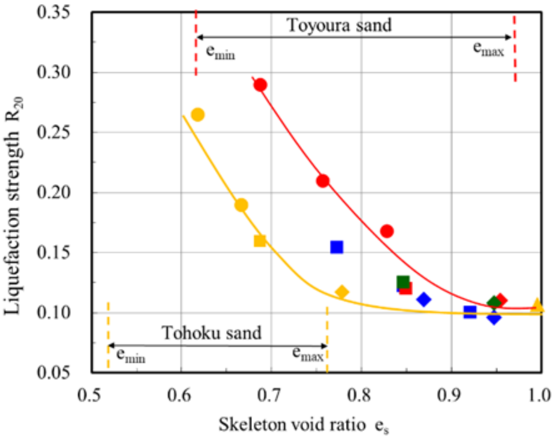


Fig. 13 The relationship between skeleton void ratio and Liquefaction strength

4.5 Relationship between Skeleton Relative Density and Liquefaction Strength

The authors also examined the skeleton relative density (Dr_{es}), which shows where the skeleton void ratio lies between the e_{min} and e_{max} of the sand. The results are shown in Eq. (2). In Fig. 12 (d), when the skeleton void ratio (e_s) is larger than the maximum void ratio e_{max} of the sand, the sand appears to float in the fine particles, and the sand particles do not interlock, resulting in a fine-grained soil skeleton. In addition, in the range where Fig. 12 (d) e_s is larger than the e_{max} of the sand, it is considered that the properties of the fine soil become stronger, so we decided to use the plasticity index of the mixed soil for the examination.

Fig. 14 shows the relationship between the relative skeleton density Dr_{es} and the liquefaction strength R_{20} of each mixed soil. The liquefaction strength also increases with increasing relative skeleton density. Although the liquefaction strength of the mixed soil with Tohoku sand, which has a larger sand grain size than Toyoura sand, tends to be higher, it falls within a unified line.

Figure 15 shows the data from previous research plotted on Figure 14. The trend line from this study is shown as a dotted line in the figure.

Although there is little comparative data, both are considered to be trends.

Skeleton relative density Dr_{es}

$$Dr_{es} = \frac{e_{max} - e_s}{e_{max} - e_{min}} \times 100 \qquad (2)$$

where, e_s : skeleton void ratio

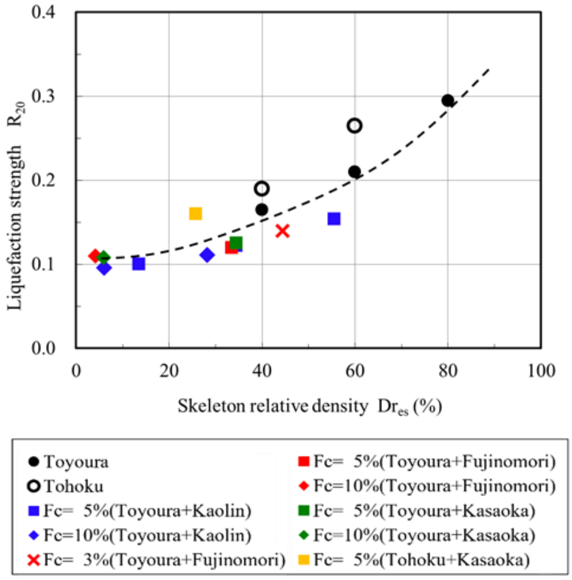


Fig. 14 The relationship between skeleton relative density and Liquefaction strength

4.6 Relationship between plasticity index and liquefaction strength

When the skeleton void ratio becomes more significant than the maximum void ratio of sand, the sand particles will lose their interlocking, and the properties of clay will become dominant. In other words, the relationship with liquefaction strength was examined using the plasticity index of the mixed soil as a parameter.

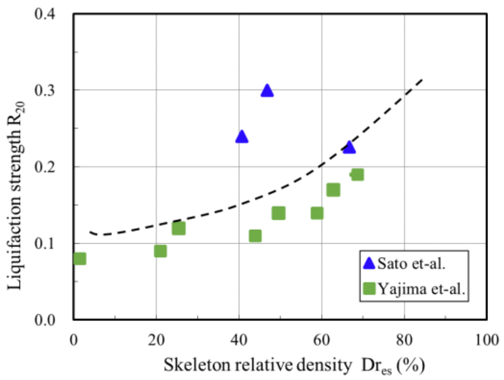


Fig. 15 The relationship between skeleton relative density and Liquefaction strength

The relationship between the plasticity index of the sand-mixed soil and liquefaction strength is shown in Fig 16. From Fig. 16, regardless of the type of sand or clay, there is a tendency for the liquefaction strength to increase as the plasticity index increases. Even with the same fine particle content, there is a tendency for the liquefaction strength to increase as the plasticity index increases.

When the soil skeleton has $e_{max} < e_s$ and the fine fraction is a non-plastic soil (NP), it was not possible to consider the skeleton relative density and/or plasticity index. In such cases, the liquefaction strength of the sandy soils was a small value, and they had almost no liquefaction strength. Further consideration is required.

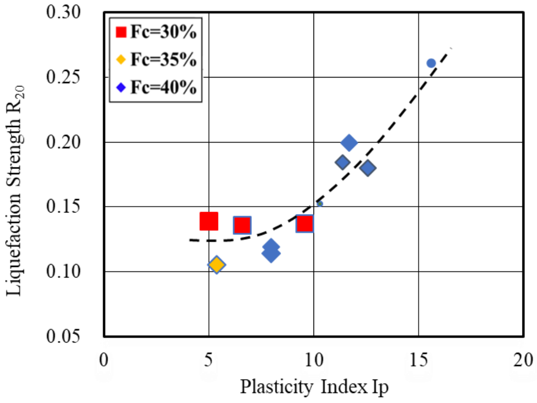


Fig. 16 The relationship between plasticity index of sand-mixed soil and Liquefaction strength

5. CONCLUSION

In this study, cyclic undrained triaxial tests were conducted on mixed soil using a standardised specimen preparation method (density control divided into 10 layers) with a fine-grained content (Fc) ranging from 0 to 60%, to evaluate the liquefaction strength of sand containing fine-grained material. Our findings can be summarized as follows.

1) While the liquefaction strength decreases with increasing skeleton void ratio, we encountered a significant challenge in evaluating the liquefaction strength based solely on the skeleton void ratio e_s. This is due to the different ranges of e_{max} and e_{min} of sand.

2) We found a significant relationship between the skeleton's relative density and liquefaction strength within the range of the minimum void ratio e_{min} of sand $\leqq$ skeleton void ratio $e_s \leqq$ maximum void ratio of sand e_{max}. This relationship is the most important in this study.

3) When the particle skeletal structure is dominated by fine fractions rather than coarse fractions, it is appropriate to evaluate the plasticity index Ip, since the properties will resemble those of the fine fractions more closely.

Future research prospects include preparing samples using the experimental results from this study with coarse-grained soils of different particle sizes and fine-grained soils with different plasticity indices, conducting cyclic undrained triaxial compression tests, and comparing liquefaction strengths. Then, the authors would like to clarify the relationships between liquefaction strength, skeletal void ratio, and skeletal relative density of sandy soil. We also plan to compare the results with those of in situ natural sediments.

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