



EFFECTS OF LONGITUDINAL AND TRANSVERSE PILE SPACING ON THE DYNAMIC IMPEDANCE OF PILED RAFT FOUNDATION

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ABSTRACT: Soil–structure interaction is crucial for evaluating the seismic response of structures. The dynamic impedance of a piled raft foundation depends on the dynamic interaction between the raft and the pile group. This paper presents a simple method for calculating the dynamic impedance of piled raft foundations with different pile spacing ratios in two directions, subjected to dynamic horizontal and rotational loads. Parametric analysis is useful for estimating system performance by varying design parameter inputs. Numerical analyses were conducted, and the obtained dynamic interactions were used to analyze frequency characteristics of the dynamic interaction factors between the raft and the pile group, which are essential for simplified calculations. The analysis revealed that, although the frequency characteristics of the dynamic interaction factor vary with the pile spacing ratio, a clear trend emerges when the pile spacing is evaluated using the geometric mean of the two directions. Furthermore, by expressing the frequency characteristics of this dynamic interaction factor as an arbitrary function, we developed a method for simplified calculation of the dynamic impedance of piled raft foundations, even when the pile spacing ratio varies. The method was calibrated for the numerical conditions investigated. The method showed good agreement within the investigated parameter range, and discrepancies increased at high frequencies. For the horizontal and rotational dynamic impedance of piled rafts within the investigated parameter range, the mean absolute percentage error for the real part was 14 %, and that for the imaginary part was 5 %.

Keywords: Piled raft, Dynamic impedance, Dynamic Interaction factor, Pile spacing, Simplified calculation

1. INTRODUCTION

A piled raft (PR) that combines a raft and piles is widely used as a building foundation that reduces settlement and improves load bearing capacity. The seismic performance of PR foundations has been researched by loading tests using centrifuge modeling and shaking tables, and by monitoring the seismic behavior of actual structures. Additionally, many researchers have applied numerical analysis to the investigation of PR foundations, due to the difficulty of conducting loading tests and a lack of monitoring data. Numerical analyses such as the finite element method [1-4], finite difference method [5, 6], models using beams on a nonlinear Winkler foundation [7], and hybrid methods combining axisymmetric finite element analysis and soil-raft-pile interaction factors [8] are frequently used to discuss the seismic behavior of the PR foundation, considering the complex soil–pile–raft interaction.

A simple numerical tool for dynamic soil–structure interaction analysis is the sway-rocking model. This model requires dynamic impedance to express horizontal and rotational motions of the

foundation. The dynamic impedance of the foundation includes dynamic stiffness and damping, which significantly affect the internal forces in the structure. Studies on the dynamic impedance of PRs have included numerical analysis, but these usually require detailed calculations [9,10]. In addition, other researchers have presented detailed analyses for the vertical dynamic impedance of a PR [11-13]. Hence, in a previous study [14], the author presented a simple method to calculate the dynamic impedance of a PR using three factors: the dynamic impedance of the spread foundation (SF), the dynamic impedance of the pile group (PG), and the dynamic interaction factor between the raft and the pile group. The dynamic interaction factor has been explored with an equation given by an arbitrary function. The amplitude and phase angle of the dynamic interaction factor was related to the static stiffness of the SF and PG, or the width of the foundation. Furthermore, the authors extended the simple method for horizontal dynamic impedance so that it can also be applied to layered soil [15]. In this research, the foundation geometry was limited to square planes. An additional analysis was conducted by varying the aspect ratio for the PRs with

rectangular planes. It was found that the frequency characteristic of the dynamic interaction factor of the rotational component changed with the aspect ratio of the foundation, and a modified formula considering the aspect ratio of the foundation was devised [16]. However, in these studies, all pile spacing was assumed to be equal in the foundation plane. When the pile spacing is different in two directions of the foundation plane, the previous equation to evaluate the dynamic interaction factor cannot be applied directly because the pile spacing was considered as a single variable in the equation.

Depending on the building column positions, the pile spacing may differ in the two directions. Different pile spacing is expected to change the effect of wave propagation from adjacent piles. The effects of pile arrangement on the impedance of the PG have been investigated as follows: It was pointed out that the frequency characteristics of the dynamic impedance of a PG strongly depend on the pile arrangement [17]. The interaction is quite different because the response is dominated by compressive waves for the piles arranged in a line and shear waves for the piles arranged side by side [18]. Even when subjected to vertical loads, the impedance of a PG is different depending on the pile configuration, such as grid, parallel line, circle, cross, and line [19]. In addition, the dynamic interaction factors were investigated by varying the raft area ratio, A_r , from 0.25 to 0.75 in a PR with four piles [20]. However, in that study, the pile spacing in the foundation plane was different in the two directions.

This paper presents a practical method for evaluating the dynamic interaction factor used to calculate the dynamic impedance of PRs with different pile spacings in two directions. The frequency characteristics of the dynamic interaction factors between the raft and the piles, which are necessary for calculation, were investigated. The frequency characteristics of the dynamic interaction factors were analyzed through a parametric study under varying pile spacing ratios and pile layouts using a detailed analysis. The results were used to develop a method of estimating the dynamic interaction factors.

2. RESEARCH SIGNIFICANCE

To evaluate the seismic response of a building supported by a PR foundation, it is essential to consider the dynamic interaction between the raft and the pile group. This paper presents a practical method for estimating the dynamic interaction factor for piled raft foundations where the different

pile spacing in two directions or where the raft-to-pile group area ratio varies. The method is intended for simplified preliminary analysis, not as a replacement for detailed numerical analysis in complex soil profiles or nonlinear seismic conditions.

3. NUMERICAL ANALYSIS

3.1 Piled Raft Model

The analytical model for the parametric study was a PR on semi-infinite and homogeneous soil, as shown in Fig. 1. The raft was modeled as massless, rigid, and with no embedded depth. The foundation comprised 25 cast-in-situ piles, with a diameter $d = 1\text{m}$ and a length $L = 25d$. The pile-raft connection was rigid. The foundation was subjected to horizontal inertial load and overturning moment due to earthquakes (Q and M , respectively). The horizontal inertial load is applied only in the x -direction shown in Fig. 1. The axis of rotation of the foundation subjected to the overturning moment becomes at the center of the foundation in the X -direction. Furthermore, when a foundation is subjected to horizontal inertial loads and overturning moments, the horizontal displacement and rotation angle of the foundation in each load direction are defined as positive.

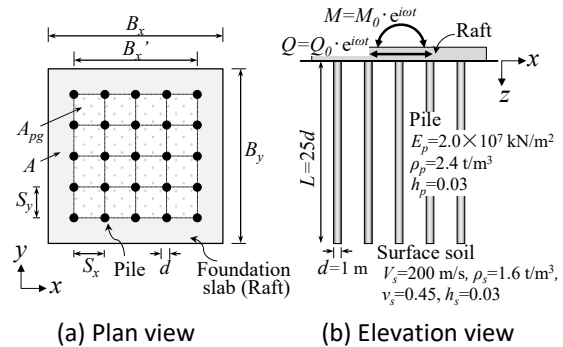


Fig. 1 Schematic of the analytical PR model

The pile and raft arrangement in PRs is expressed as the ratio of pile spacing in the two directions, S_{xy} ($= S_x/S_y$), and the ratio of the area circumscribed by the PG to the total raft area, A_r ($= A_{pg}/A$), as shown in Fig. 2. Five conditions were set for the pile spacing ratio ($S_{xy} = 0.50, 0.67, 1.00, 1.50, 2.00$), and three conditions were set for the pile layout ($A_r = 0.25, 0.50, 1.00$). When A_r changes, the raft widths B_x and B_y both change by a factor of $A_r^{0.5}$. The pile group remains centered under the raft. This created a total of 15 analysis cases, as shown in Table 1.

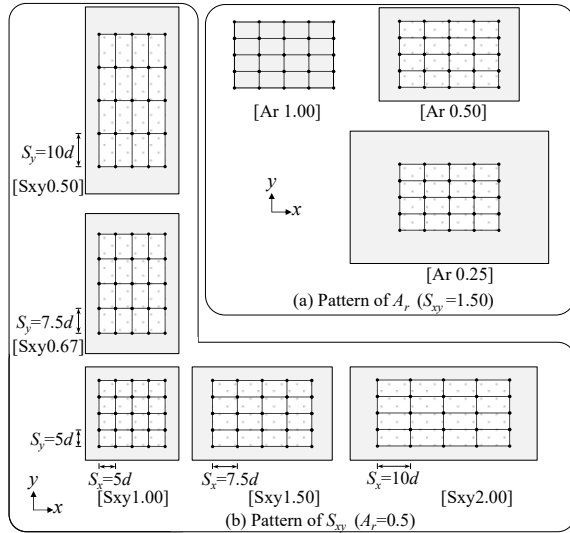


Fig. 2 Pile and raft arrangements

Table 1 List of analysis parameters

Case	A_r	$S_{xy} (S_x : S_y)$	B_x, B_y (m)
1	1.00	0.50 (5:10)	20, 40
2	1.00	0.67 (5: 7.5)	20, 30
3	1.00	1.00 (5:5)	20, 20
4	1.00	1.50 (7.5:5)	30, 20
5	1.00	2.00 (10:5)	40, 20
6	0.50	0.50 (5:10)	28.28, 56.57
7	0.50	0.67 (5: 7.5)	28.28, 42.43
8	0.50	1.00 (5:5)	28.28, 28.28
9	0.50	1.50 (7.5:5)	42.43, 28.28
10	0.50	2.00 (10:5)	56.57, 28.28
11	0.25	0.50 (5:10)	40, 80
12	0.25	0.67 (5: 7.5)	40, 60
13	0.25	1.00 (5:5)	40, 40
14	0.25	1.50 (7.5:5)	60, 40
15	0.25	2.00 (10:5)	80, 40

Note: The analysis parameters for No. 3 are the same as those in the previous study [14].

3.2 Methodology

The Main step of the research process in this paper is shown in Fig. 3.

First, a parametric study was conducted by varying the pile spacing ratio and the pile layout to obtain the dynamic impedance of the PRs and to investigate the dynamic interaction factors between the raft and the piles.

The numerical analysis for a previous parametric study used a hybrid method that combines the finite element method and the thin layer method. The details regarding the equation of motion for the foundation–soil system in the frequency domain in

the hybrid FEM–thin layer method and the element stiffness matrices in the FEM elements such as raft, piles, and pile head connections are described in detail in a previous study [14]. In the finite element analysis, the raft was modeled as a thin plate element and the pile was modeled as a beam element. The ground motion was represented by three-dimensional frequency-domain Green's functions for a homogeneous half-space with the thin layer method. The pile elements were divided into 50 equal segments along their length. The mesh convergence study for this analysis was conducted to verify the dynamic response of the foundation. The dynamic stiffness of each material was assumed to be that of a linear viscoelastic model consisting of an elastic spring and a viscous dashpot in parallel. To introduce hysteretic material damping, the real elastic modulus is multiplied by $1+2hi$ (where h is the damping ratio), making it complex. However, as this is a linear viscoelastic model, the method has its limits when applied to actual nonlinear conditions of the soil.

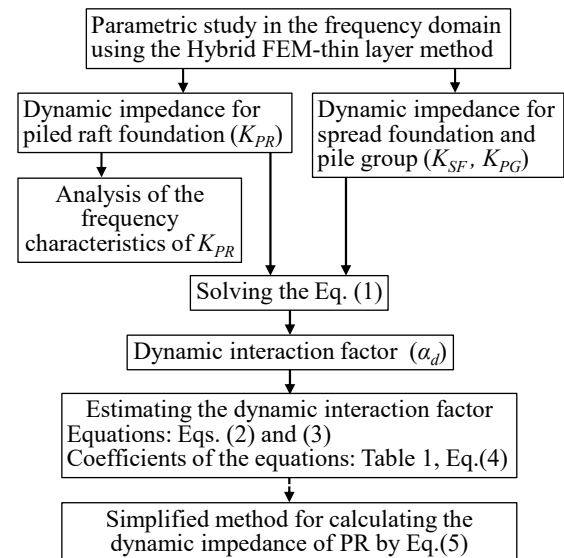


Fig. 3 Main steps of the research process

A frequency response analysis was performed to calculate the dynamic impedance exerted on the foundations by the horizontal inertial forces or overturning moments caused by earthquakes. Dynamic loads acting on the foundation were assumed to be these horizontal or moments of the building. The harmonic load was applied at frequencies ranging from $f = 1$ to 40 Hz, in increments of $\Delta f = 1$ Hz. The dynamic displacements in the horizontal and rotational directions were

obtained from the analysis.

The dynamic impedance was calculated by dividing the dynamic load acting on the foundation by the dynamic displacement occurring in the foundation.

The dynamic interaction factor α_d was obtained by substituting the dynamic impedances (K_{PR} , K_{SF} , K_{PG}) of three foundation configurations (the SF, where the dimensions of the foundation slab are equal to the PR; the PG, where the lengths of the piles are the same as the PR; and the PR) in each case, into Eq. (1) and solving it. The dynamic interaction factor represents the dynamic interaction between the PG and the raft. The dynamic interaction factor is meant to transform the magnitude and initial angle of the displacement component due to the total load carried by the group piles of the PR.

$$\frac{K_{PR}(f)}{K_{PG}(f)} \alpha_d^2(f) - 2\alpha_d(f) + \frac{K_{PG}(f)}{K_{SF}(f)} - \frac{K_{PR}(f)}{K_{SF}(f)} + 1 = 0 \quad (1)$$

4. DYNAMIC INTERACTION FACTOR

4.1 Analysis Results and Considerations

The dynamic interaction factor was treated as a complex number expressed as $\alpha_d [=|\alpha| \cdot \text{Exp}(i\varphi_a)]$, where $|\alpha|$ and φ_a represent the amplitude and phase angle of the dynamic interaction factor, respectively. The variation of dynamic interaction factors for horizontal motion of PRs with different pile spacings for $A_r = 0.5$ is shown in Figs. 4 and 5. When $S_{xy} > 1$, the phase angle gradually increases as with $S_{xy} = 1$. In contrast, when $S_{xy} < 1$, the amplitude appears to have two peaks, and the phase angle tends to decrease temporarily at an arbitrary frequency. Because the propagation velocity of waves from adjacent piles is slower in the Y direction than in the X direction, we conclude that an out-of-phase condition occurs when $S_{xy} < 1$.

The variation of dynamic interaction factors for horizontal and rotational motion of PRs with different raft areas for $S_{xy} = 0.5$ is shown in Figs. 6 and 7. Changes in the A_r value have little effect on the phase angle, and this trend is particularly pronounced in horizontal motion. Similar results for the amplitude of the dynamic interaction factor decreasing as A_r becomes smaller have been obtained by the previous study [20].

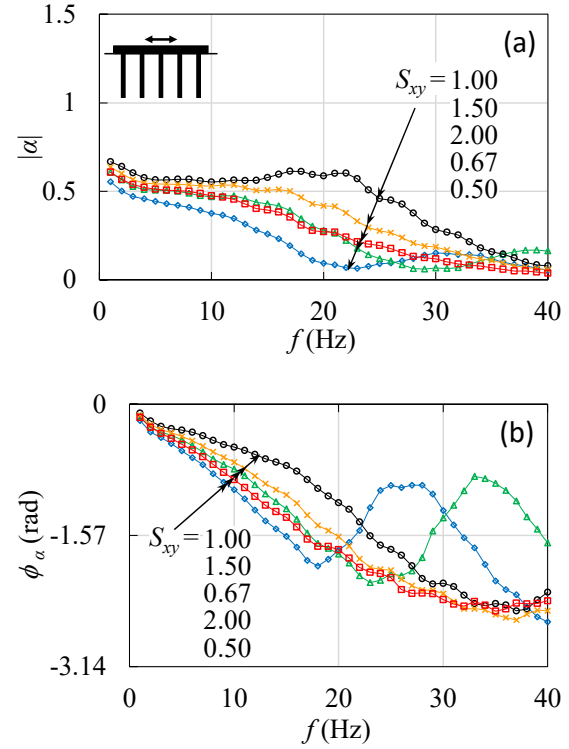


Fig. 4 Dynamic interaction factors for horizontal motion of PRs with different pile spacing for $A_r = 0.5$ (Cases 6–10): (a) amplitude, (b) phase angle

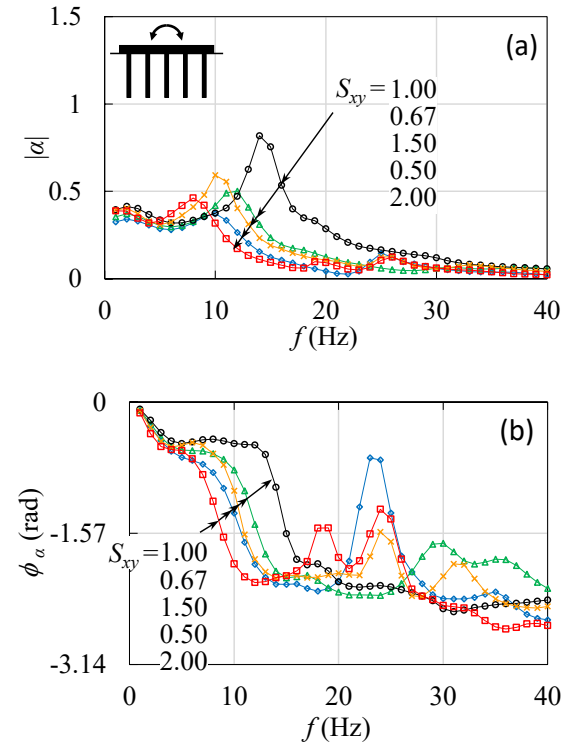


Fig. 5 Dynamic interaction factors for rotational motion of PRs with different pile spacing for $A_r = 0.5$ (Cases 6–10): (a) amplitude, (b) phase angle

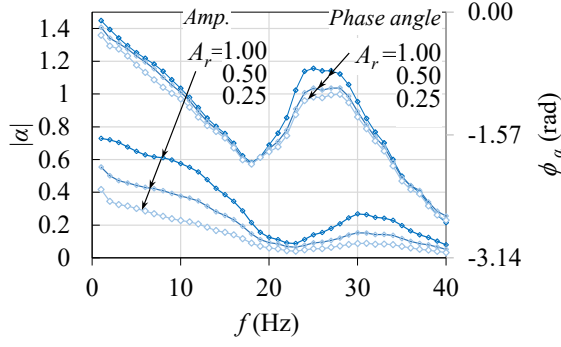


Fig. 6 Dynamic interaction factors for horizontal motion of PRs with different raft areas for $S_{xy} = 0.5$ (Cases 1, 6, 11)

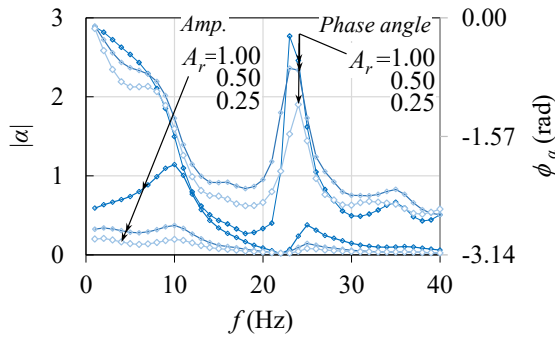


Fig. 7 Dynamic interaction factors for rotational motion of PRs with different raft areas for $S_{xy} = 0.5$ (Cases 1, 6, 11)

4.2 Estimation of Dynamic Interaction Factor

The frequency characteristics of the amplitude and the phase angle of the dynamic interaction factor were estimated using Eqs. (2) and (3), which

are extensions of equations from previous studies [14]. These equations consider the contributions of two natural modes of vibration to the amplitude and phase: the dynamic interaction factor reflects the presence of two amplitude peaks and the tendency of the phase angle to decrease temporarily at any given frequency.

$$|α(a)| = \xi \cdot \frac{a_{i1}^2 \cdot a_{i2}^2}{(1 - \beta) \cdot a_{i1}^2 + \beta \cdot a_{i2}^2} \cdot \exp(-\xi a) \cdot \sqrt{(\chi_{11} + \chi_{21})^2 + (\chi_{12} + \chi_{22})^2} \quad (2)$$

$$\phi_a(a) = -\tan^{-1} \left(\frac{\chi_{11} + \chi_{21}}{\chi_{12} + \chi_{22}} \right) \quad (3)$$

where

$$\chi_{11} = \frac{\eta_1 \cdot a_{i1} \cdot a}{(\eta_1 \cdot a_{i1} \cdot a)^2 + (a_{i1}^2 - a^2)^2} \cdot \beta$$

$$\chi_{12} = \frac{a_{i1}^2 - a^2}{(\eta_1 \cdot a_{i1} \cdot a)^2 + (a_{i1}^2 - a^2)^2} \cdot \beta$$

$$\chi_{21} = \frac{\eta_2 \cdot a_{i2} \cdot a}{(\eta_2 \cdot a_{i2} \cdot a)^2 + (a_{i2}^2 - a^2)^2} \cdot (1 - \beta)$$

$$\chi_{22} = \frac{a_{i2}^2 - a^2}{(\eta_2 \cdot a_{i2} \cdot a)^2 + (a_{i2}^2 - a^2)^2} \cdot (1 - \beta)$$

The coefficients determined for the estimation of the dynamic interaction factor are summarized in Table 2. The method used to determine these coefficients is described below.

Table 2 Coefficients for estimating the dynamic interaction factor

	Horizontal		Rotational	
	$S_{xy} \leq 1.0$	$S_{xy} > 1.0$	$S_{xy} \leq 1.0$	$S_{xy} > 1.0$
ξ_0	$\frac{ K_{PR} }{ K_{SF} } = \frac{1}{7.5} \left(\frac{ K_{PG} }{ K_{SF} } \right)^{3.30} + 1$		$\frac{ K_{PR} }{ K_{SF} } = \frac{1}{2.7} \left(\frac{ K_{PG} }{ K_{SF} } \right)^{1.57} + 1$	
η_1	$\frac{A_r}{1.50 \left(\frac{ K_{PG} }{ K_{SF} } \right)^2 - 0.33 \left(\frac{ K_{PG} }{ K_{SF} } \right)}$		$\frac{A_r}{0.24 \left(\frac{ K_{PG} }{ K_{SF} } \right)^2 + 1.05 \left(\frac{ K_{PG} }{ K_{SF} } \right)}$	
λ	$\frac{0.66}{S_{xy} \sqrt{A_r}}$		0.66	
a_{i1}	0.56		$\frac{1}{B_x^{0.33}}$	
λ	0.2		0.2	
k	$0.50 \cdot S_{xy} + 1.75$	$1.25 \cdot S_{xy} + 1.00$	$3.0 \cdot S_{xy} + 1.0$	$-1.5S_{xy} + 5.5$
β	$0.8 \cdot S_{xy}$	0.8	0.67	

Note: $\eta_2 = \lambda \cdot \eta_1$, and $a_{i2} = \kappa \cdot a_{i1}$

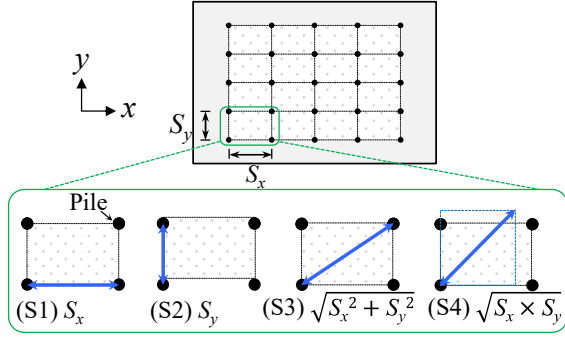


Fig. 8 Use of pile spacing in the calculation of dimensionless frequency

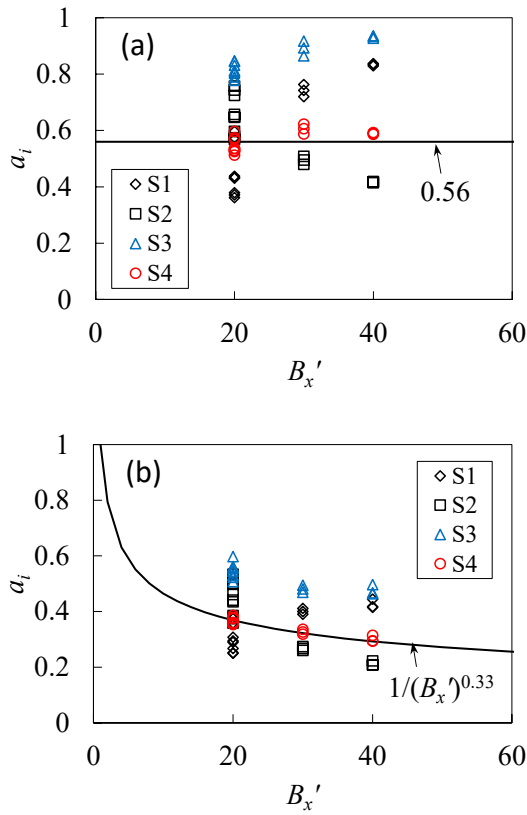


Fig. 9 Correlation between the effective width of foundation B_x' and coefficient a_i at different pile spacings: (a) horizontal, (b) rotational

Parameter a is the dimensionless frequency, which is defined in terms of a single pile spacing parameter S , representing the pile spacing, as shown in Eq. (4)[14].

$$a = \frac{fS}{V_s} \quad (4)$$

where f is the frequency, S is the pile spacing, and V_s is the shear wave velocity of the soil.

However, in this analytical model, the pile spacing differs in the two directions of the foundation plane. Therefore, we defined a dimensionless frequency by replacing the two pile spacings with a single pile spacing, as shown in Fig. 8, and investigated whether coefficient a_{i1} could be expressed using the previous calculation formula [14].

The correlation between the effective width of the foundation, $B_x' [=(\sqrt{n}-1) \times S_x]$, and coefficient a_i calculated by different concepts of pile spacing is shown in Fig. 9. In both the horizontal and rotational components, when the pile spacing was represented with the geometric mean value in two directions, $S = \sqrt{S_x \times S_y}$, the result was consistent with the approximated value.

Coefficients ξ , η_1 , η_2 , ζ , a_{i1} , a_{i2} , and θ were set to fit the dynamic interaction factor obtained by a parametric study using the numerical analysis.

Coefficient ξ and coefficient ζ for the rotational component are described in previous studies [14,16].

These coefficients are related to the stiffness ratio $|K_{PR}|/|K_{SF}|$ of the PG to the SF at a frequency of $f = 0.1$ Hz, which is close to the static state. Coefficient ξ is calculated by substituting $|K_{PR}|$ and $|K_{PG}|/|K_{SF}|$ into Eq. (1). Then, $|K_{PR}|$ is obtained from the correlation with $|K_{PG}|/|K_{SF}|$ using the equation for ξ_0 given in Table 2. Coefficient η_1 was newly approximated as a function of $|K_{PR}|/|K_{SF}|$, as shown in Fig. 10.

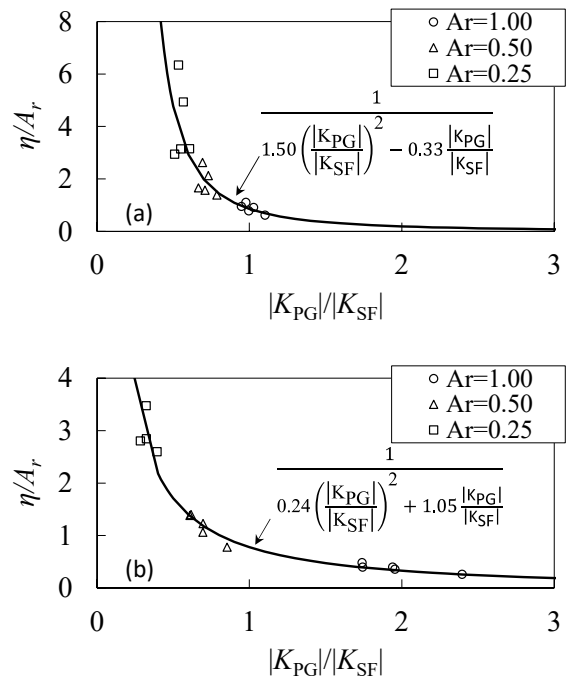


Fig.10 Correlation between the stiffness ratio $|K_{PG}|/|K_{SF}|$ of the PG to the SF and coefficient η/Ar : (a) horizontal, (b) rotational

It was decided to express coefficients η_2 and a_{i2} as λ times η_1 and κ times a_{i1} , respectively. Coefficients a_{i2} , η_2 , and θ were fitted as functions of S_{xy} and A_r , as shown in Figs. 11–13, to match the dynamic interaction factors obtained from the parametric study using numerical analysis.

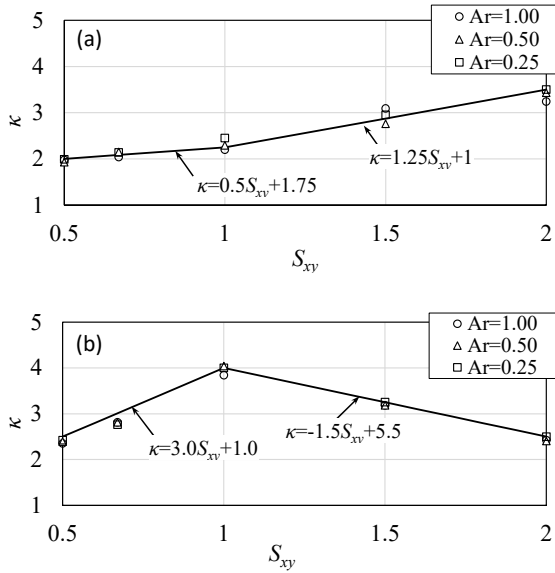


Fig. 11 Correlation between the pile spacing ratio and coefficient κ : (a) horizontal, (b) rotational

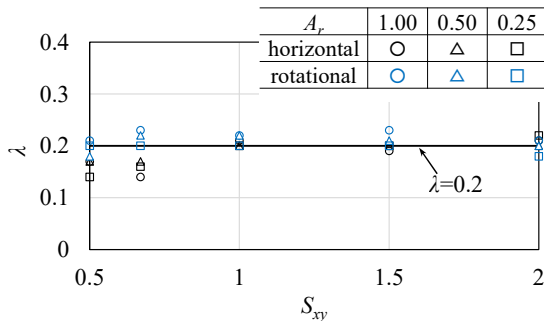


Fig. 12 Correlation between the pile spacing ratio and coefficient λ

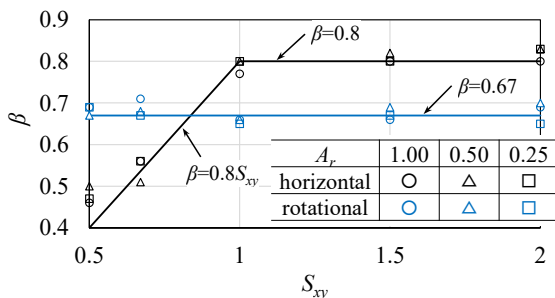


Fig. 13 Correlation between the pile spacing ratio and coefficient θ

The dynamic interaction factors obtained from analysis and calculation for Cases 6 and 10 are compared in Fig. 14. Although the calculated results for horizontal and rotational dynamic interaction factors may show slight discrepancies, the calculated results represent the frequency characteristics obtained from the analysis.

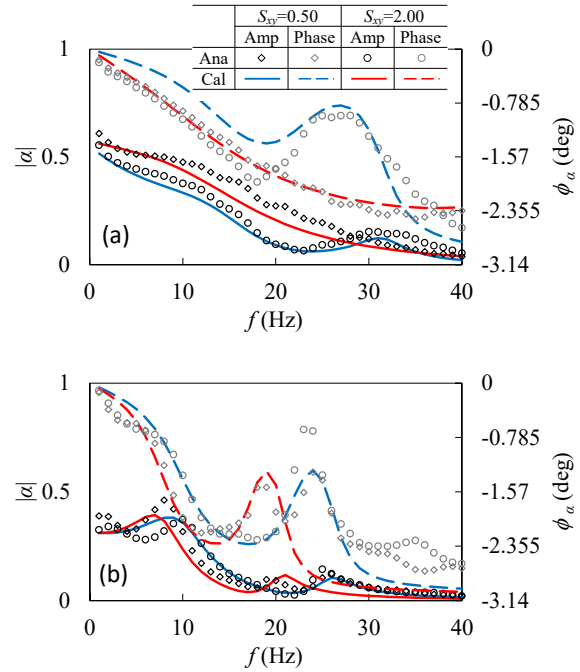


Fig. 14 Comparison between analysis and calculation results for the dynamic interaction factors in Cases 6 and 10: (a) horizontal, (b) rotational

5. DYNAMIC IMPEDANCE

We used the dynamic interaction factor formulated above to confirm the applicability of the simplified method for calculating the dynamic impedance of PR foundations.

5.1 Simplified Method for Calculating Dynamic Impedance

In the simplified method, the dynamic impedance of the PR, K_{PR} , is calculated using Eq. (5). Equation (5) was derived by extending the approach to a static problem for piled raft foundations [21] to a dynamic problem. This equation is derived from the three relationships that compose the PR: the SF, the PG, and the raft–piles interaction [14]. The equation requires three variables: the dynamic impedances of the SF and the PG, K_{SF} and K_{PG} , respectively, and the dynamic interaction factor, α_d , between the raft and the pile group.

In this study, the dynamic impedance of the SF and that of the PG used in the simplified method were calculated using values derived from the numerical analysis, to clarify the applicability of the proposed dynamic interaction factor.

$$K_{PR}(f) = \frac{K_{PG}(f) + \{1 - 2\alpha_d(f)\}K_{SF}(f)}{1 - \alpha_d^2(f)\{K_{SF}(f)/K_{PG}(f)\}} \quad (5)$$

5.2 Comparison between Simplified Calculation and Numerical Analysis

The dynamic impedance values obtained from analysis and calculation for cases 6 and 10 are compared in Fig. 15. The calculated results are in close agreement with the analysis results. For the horizontal and rotational dynamic impedance of piled rafts, the mean absolute percentage error (MAPE) for the real part of the calculated values relative to the analytical values was 13–16 %, and that for the imaginary part was 2–6 %. The factor contributing to the decline in calculation accuracy with increasing frequency is the limitation of the proposed formula for the dynamic interaction factor. Specifically, whilst the proposed formula can represent up to two peaks arising from the amplitude of the dynamic interaction factor, there are cases where three or more peaks appear, and the formula was unable to represent these. To estimate the high-precision dynamic impedance of the PR at high frequencies, it will be necessary to carry out the detailed numerical analysis.

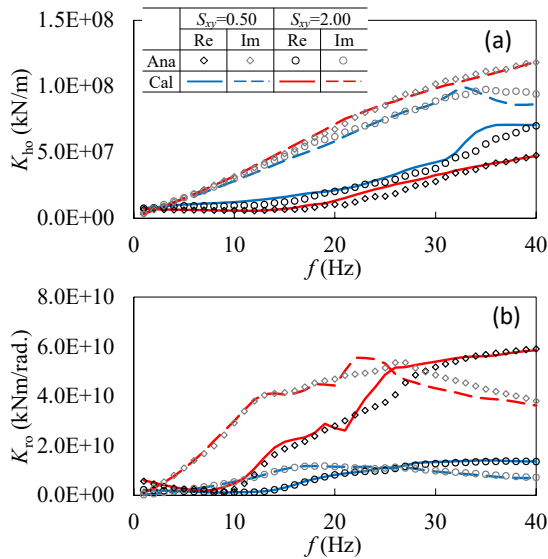


Fig. 15 Comparison between analysis and calculation results for the dynamic impedances in Cases 6 and 10: (a) horizontal, (b) rotational (Re = real; Im = imaginary)

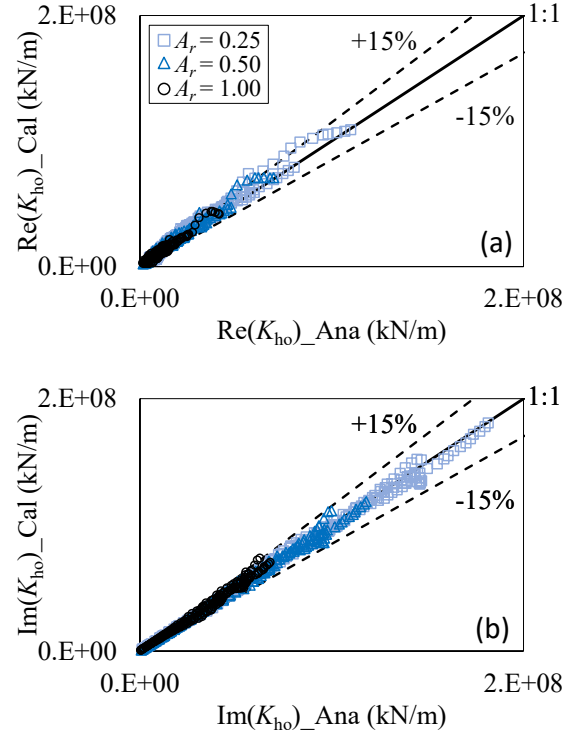


Fig. 16 Comparison of horizontal dynamic impedances between calculation and analysis results in all cases: (a) real part, (b) imaginary part

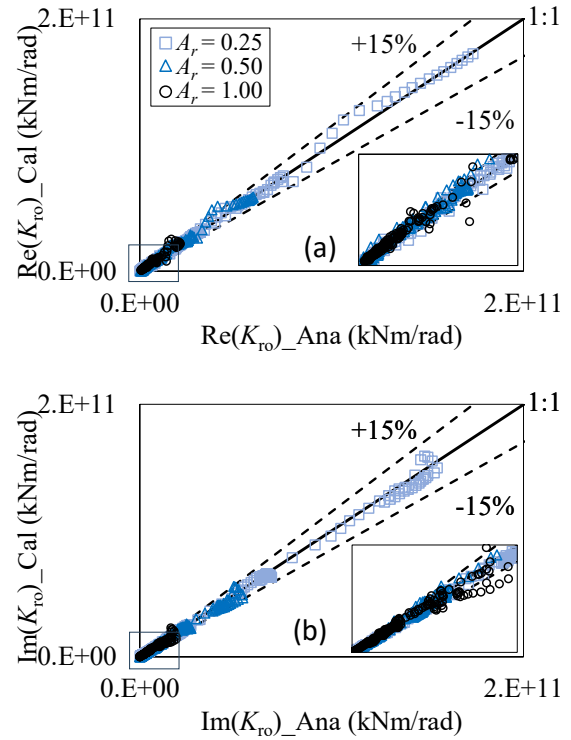


Fig. 17 Comparison of rotational dynamic impedances between calculation and analysis results in all cases: (a) real part, (b) imaginary part

Figs. 16 and 17 show a comparison of the calculated results (Cal) and the analyzed results (Ana) of the dynamic impedance of the PRs in all cases [where $K_m = |K_m| \cdot \exp(i\varphi_m)$ and m is an index (ho: horizontal, ro: rotation)]. The calculated value agrees with the analyzed value. It seems that the smaller A_r , the closer the calculated value is to the analytical value. Because the frequency characteristic of the dynamic impedance of the PRs becomes similar to the state of the spread foundation, the influence of the dynamic interaction factor becomes small. For the horizontal and rotational dynamic impedance of piled rafts within the investigated parameter range, the MAPE for the real part was 14 %, and that for the imaginary part was 5 %. As the real part of the calculated dynamic impedance has been overestimated by approximately 10 % compared with the analytical value, the MAPE for the real part of the calculated dynamic impedance is slightly higher than that for the imaginary part. Furthermore, the proportion of calculated values falling within ± 15 % of the analytical values was 60% for the real part and 99% for the imaginary part for horizontal dynamic impedances. In addition, for rotational dynamic impedances, the proportion was 80% for the real part and 90% for the imaginary part.

A slight discrepancy between the calculated and analytical values is observed in the results for high frequencies. Consequently, care must be taken when applying the simplified calculation method for the dynamic impedance at high frequencies in cases where the pile spacing ratio in the PR differs significantly.

6. CONCLUSION

This study investigated the dynamic impedance and dynamic interaction factor of PR foundations with different pile spacings in two directions, subjected to horizontal and rotational loads. The results are summarized below:

(1) The dynamic impedance and the dynamic interaction coefficient were significantly affected by the ratio of the pile spacing in the two directions.

(2) It has been found that the geometric mean is an effective equivalent pile spacing for the dimensionless frequency when evaluating the dynamic interaction factor of the piled raft foundation with different pile spacings in two directions.

(3) The simplified method for calculating the dynamic impedance of the piled raft foundation showed good agreement within the investigated

parameter range, and discrepancies increased at high frequencies.

Although the presented simple method has the limit of application described above, it has become applicable to piled raft foundations supporting buildings with more general floor plans, where the column spans differ between the longitudinal and transverse directions.

Furthermore, we plan to address piled raft foundations with random pile arrangements or multiple pile dimensions in the future.

7. REFERENCES

1. Azizkandi A.S., Baziar M.H., and Yeznabad A.F., 3D Dynamic Finite Element Analyses and 1 g Shaking Table Tests on Seismic Performance of Connected and Nonconnected Piled Raft Foundations, *KSCE Journal of Civil Engineering*, Vol. 22, Issue 5, 2018, pp. 1750-1762. <https://doi.org/10.1007/s12205-017-0379-2>
2. Chaudhuri C.H., Chanda D., Saha R., and Haldar S., Three-dimensional numerical analysis on seismic behavior of soil-piled raft-structure system, *Structures*, Vol. 28, 2020, pp. 905-922. <https://doi.org/10.1016/j.istruc.2020.09.024>
3. Patil G., Choudhury D., and Mondal A., Interactive iterative methodology for seismic analysis of pile-raft supported nuclear power plant structure in a conventional framework, *Soil Dynamics and Earthquake Engineering*, Vol. 173, 2023, article 108109, pp. 1-12. <https://doi.org/10.1016/j.soildyn.2023.108109>
4. Swasdi S., Chub-Uppakarn T., Srisakul W., Panedpojaman P., and Sae-Long W., Enhancing piled raft foundation performance in clay: Exploring the influence of cyclic lateral loading patterns, frequency, and cycles, *Ocean Engineering*, Vol. 327, 2025, article 120977, pp. 1-24. <https://doi.org/10.1016/j.oceaneng.2025.120977>
5. Bazaz H.B., Akhtarpour A., and Karamodin A., A study on the effects of piled-raft foundations on the seismic response of a high rise building resting on clayey soil, *Soil Dynamics and Earthquake Engineering*, Vol. 145, 2021, article 106712, pp. 1-10. <https://doi.org/10.1016/j.soildyn.2021.106712>
6. Maheshwari B.K., and Firoj M., Seismic response of combined piled raft foundation using advanced liquefaction model, *Soil Dynamics and Earthquake Engineering*, Vol. 181, 2024, article 108694, pp. 1-14. <https://doi.org/10.1016/j.soildyn.2024.108694>

7. Saha R., Dutta S.C., Halder S., and Kumar S., Effect of soil-pile raft-structure interaction on elastic and inelastic seismic behaviour, *Structures*, Vol. 26, 2020, pp. 378-395.
<https://doi.org/10.1016/j.istruc.2020.04.022>
8. Bernardes H.C., Carvalho S.L., Sales M.M., Almeida S.R.M., Farias M.M., and Pinho F.A.X.C., Hybrid numerical tool for nonlinear analysis of piled rafts, *Soils and Foundations*, Vol. 59, Issue 6, 2019, pp. 1659-1674.
<https://doi.org/10.1016/j.sandf.2019.04.011>
9. Labaki J., Barros P.L.A., and Mesquita E., Coupled horizontal and rocking vibration, and seismic shear-wave scattering of a piled plate on a transversely isotropic half-space, *Engineering Analysis with Boundary Elements*, Vol. 106, 2019, pp. 609-623.
<https://doi.org/10.1016/j.enganabound.2019.04.016>
10. Firoj M., and Maheshwari B.K., Dynamic impedances of CPRF using coupled BEM-FEM approach: A parametric study and application, *Engineering Analysis with Boundary Elements*, Vol. 156, 2023, pp. 8-19.
<https://doi.org/10.1016/j.enganabound.2023.07.036>
11. Nguyen V.T., Hassen G., and de Buhan P., Assessing the dynamic stiffness of piled-raft foundations by means of a multiphase model, *Computers and Geotechnics*, Vol. 71, 2016, pp. 124-135.
<https://doi.org/10.1016/j.compgeo.2015.08.014>
12. Liu C.L., and Ai Z.Y., Vertical harmonic vibration of piled raft foundations in layered soils, *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 41, Issue 17, 2017, pp. 1711-1723.
<https://doi.org/10.1002/nag.2696>
13. Barros P.L.A., Labaki J., and Mesquita E., IBEM-FEM model of a piled plate within a transversely isotropic half-space, *Engineering Analysis with Boundary Elements*, Vol. 101, 2019, pp. 281-296.
<https://doi.org/10.1016/j.enganabound.2018.12.013>
14. Nagai H., Simplified method of estimating the dynamic impedance of a piled raft foundation subjected to inertial loading due to an earthquake. *Computers and Geotechnics*, Vol. 105, 2019, pp. 69-78.
<https://doi.org/10.1016/j.compgeo.2018.09.014>
15. Nagai H., Takai T., and Yasuda Y., Simplified calculation of horizontal dynamic impedance of piled raft foundation on layered soil, *Geotechnical and Geological Engineering*, Vol. 42, Issue 11, 2024, pp. 7835-7846.
<https://doi.org/10.1007/s10706-024-02953-8>
16. Nagai H., Evaluation of dynamic interaction factor of rectangular piled raft foundation for horizontal and rotational motion, *International Journal of GEOMATE*, Vol. 20, Issue 77, 2021, pp. 107-115.
<https://doi.org/10.21660/2020.77.28817>
17. Choudhary S.S., Biswas S., and Manna B., Effect of pile arrangements on the dynamic coupled response of pile groups, *Geotechnical and Geological Engineering*, Vol. 39, 2021, pp. 1963-1978.
<https://doi.org/10.1007/s10706-020-01599-6>
18. Luan L., Zheng C., Kouretzis G., and Ding X., Dynamic analysis of pile groups subjected to horizontal loads considering coupled pile-to-pile interaction, *Computers and Geotechnics*, Vol. 117, 2020, article 103276, pp. 1-8.
<https://doi.org/10.1016/j.compgeo.2019.103276>
19. Auersch L., Wave propagation in the elastic half-space due to an interior load and its application to ground vibration problems and building on pile foundations, *Soil Dynamic and Earthquake Engineering*, Vol. 30, Issue 10, 2010, pp. 925-936.
<https://doi.org/10.1016/j.soildyn.2010.04.003>
20. Varghese R., Boominathan A., and Banerjee S., Stiffness and load sharing characteristics of piled raft foundations subjected to dynamic loads, *Soil Dynamics and Earthquake Engineering*, Vol. 133, 2020, pp. 106-117.
<https://doi.org/10.1016/j.soildyn.2020.106117>
21. Clancy P., and Randolph M.F., An approximate analysis procedure for piled raft foundations. *International Journal Numerical and Analytical Methods in Geomechanics* Vol. 17, 1993, pp. 849-69.
<https://doi.org/10.1002/nag.1610171203>

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