

# ENHANCING SOIL ELASTIC MODULUS ESTIMATION THROUGH DERIVATIVE CALCULATIONS AND CONSOLIDATION TEST

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**ABSTRACT:** This study presents a novel approach to determining the soil's elastic modulus ( $E'$ ), a critical parameter in geotechnical engineering, by employing derivative-based methodologies and one-dimensional consolidation test results. The main challenges of traditional methods, such as triaxial CD testing, include high costs, long duration, and complexity in data collection and analysis. This new approach addresses these challenges by applying derivative calculations at a specific reference stress point ( $P_{ref}$ ), resulting in a tangent equation that accurately represents the soil's compressive behavior. Utilizing the results from one-dimensional consolidation tests not only reduces dependence on costly triaxial CD tests but also ensures high accuracy in evaluating soil mechanical properties. The findings indicate that the  $E'$  value obtained from this new method is equivalent to that from triaxial CD tests, confirming the method's feasibility and effectiveness. The unique achievement of this research is the development of a fast, cost-effective, and efficient method for determining the soil's elastic modulus, opening new research directions in the field of geotechnical engineering.

*Keywords: Elastic modulus, Derivative methodologies, Consolidation test, Cost-effective, Soil analysis*

## 1. INTRODUCTION

Determining the soil's elastic modulus ( $E'$ ) has always been a significant challenge in geotechnical engineering, especially when using traditional methods like triaxial CD testing, which require high costs and long durations [1-4]. One of the main challenges is the complexity of accurately collecting and analyzing data from these tests, leading to the search for more efficient alternative methods [5, 6]. This study has achieved a unique breakthrough by applying derivative calculations at a specific reference stress point ( $P_{ref}$ ), creating a tangent equation that accurately models the soil's compressive behavior at  $P_{ref}$ . This method not only simplifies the process but also significantly reduces costs and time compared to traditional methods.

The standout difference of this study compared to previous works is the use of one-dimensional consolidation test results combined with derivative calculations to determine the soil's elastic modulus. While previous studies primarily relied on direct measurements from triaxial CD tests and non-traditional techniques that incur very high costs [7-13], this study demonstrates that data from one-dimensional consolidation tests can be effectively used as a substitute without compromising the accuracy of the results. This combination not only reduces dependence on costly triaxial CD tests but also provides a quick and cost-effective method for determining the soil's elastic modulus ( $E'$ ).

The research process began with analyzing soil

behavior under various loading stages during consolidation. Data collected from these tests were then used to extrapolate the elastic modulus ( $E'$ ) values and compare them with values obtained from traditional triaxial tests and the formula for calculating the elastic modulus ( $E'$ ) by Tassios and Anagnostopoulos from Standard Penetration Test (SPT) results. This analysis showed a high degree of correlation, confirming the feasibility and accuracy of the proposed method. This indicates that derivative calculations and results from one-dimensional consolidation tests can be effectively used to determine the soil's elastic modulus, opening up new research directions and approaches in the field of geotechnical engineering.

The structure of this research paper includes several main sections. After the introduction, the paper highlights the significance of the research, focusing on the innovation of the proposed method. The materials and methods section details the experimental features and research methodologies used. Specifically, the experimental setups and procedures applied to ensure the accuracy and reliability of the results are explained. The one-dimensional consolidation tests and triaxial CD tests are clearly presented, along with the method of applying derivative calculations at the reference stress point ( $P_{ref}$ ) to develop a tangent equation that models the soil's compressive behavior.

The results section presents the findings from the experiments and data analysis, including the elastic modulus ( $E'$ ) values obtained from the experiments

and their comparison with values from traditional triaxial tests. The discussion section delves into the analysis and interpretation of the results, comparing them with previous studies and practical test outcomes.

The wide applications of this study are significant in various fields of geotechnical engineering. In the field of infrastructure construction, the new method can improve design and optimize costs for projects such as roads, bridges, dams, and other civil structures. By providing a quick and accurate method for determining the soil's elastic modulus, this research aids in the rapid assessment of soil conditions, allowing timely interventions in construction projects. Furthermore, this method can be used for quality control of foundation soils, enhancing the accuracy and efficiency of geotechnical evaluations, thereby contributing to the safety and quality assurance of construction works.

## 2. RESEARCH SIGNIFICANCE

This study introduces an innovative approach using derivative calculations and results from one-dimensional consolidation tests to estimate  $E'$ . This approach not only decreases the need for extensive triaxial CD testing but also offers a cost-effective and time-efficient method for geotechnical evaluation. Our findings suggest that derivative calculations and one-dimensional consolidation test results are viable means for determining soil's  $E'$ , paving the way for a new approach in soil mechanics analysis without the need for expensive and time-consuming traditional testing methods. This study not only brings new insights into the elastic behavior of soil but also proposes an economical and effective solution.

## 3. MATERIAL AND METHOD

### 3.1 Experimental Features

#### 3.1.1 One-dimensional consolidation test characteristics

The one-dimensional consolidation test is crucial in geotechnical engineering as it offers vital information about the mechanical changes that occur in soil when subjected to compressive forces [14-17]. These tests generate critical data on the deformation modulus, an indispensable index for evaluating soil's mechanical properties. The process begins with the careful selection of soil samples, representing the terrain under investigation accurately. Following sample preparation, specialized equipment applies precise compressive forces, capturing data on soil transformation through embedded pressure and deformation sensors. This meticulous approach ensures the reliability and repeatability of the test results, offering a practical overview of soil behavior under varying load conditions.

#### 3.1.2 Triaxial CD test characteristics

The triaxial CD test, known for its accuracy in evaluating soil's mechanical properties, demands significant time and financial investments [18-24]. It assesses the soil's behavior under controlled drained conditions, providing valuable information on its strength and stiffness parameters. By subjecting the soil sample to axial stress while allowing lateral drainage, the test simulates natural soil conditions, ensuring the reliability of the results. Despite its advantages, the test's resource-intensive nature has prompted the search for alternative methodologies that can offer similar insights more efficiently.

## 3.2 Research Methodology

A basic idea for the hyperbolic relationship between the vertical strain,  $\epsilon$  and the deviatoric stress  $q$ , in primary triaxial loading. This technique employs derivative calculations and results from one-dimensional consolidation tests, providing a stark contrast to the traditional triaxial CD tests which are both time-intensive and costly.

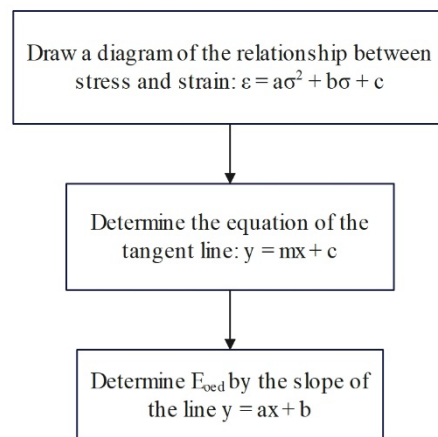


Fig. 1 General procedure for determining  $E_{oeed}$

Our researchers have applied derivatives to the stress and deformation relationship at a specific reference stress point  $P_{ref}$  to develop a tangent equation that accurately represents soil compression behavior. This novel approach simplifies the testing process, providing a quick assessment tool that greatly saves resources. Moreover, by analyzing data from various stages of one-dimensional consolidation tests, this methodology facilitates the calculation of the soil's elastic modulus ( $E'$ ), demonstrating the utility of consolidation tests in assessing soil stiffness and reducing reliance on extensive triaxial testing. The general procedure for determining  $E_{oeed}$  (Fig. 1).

In consolidation compression experiments, the relationship between stress and deformation is represented by an anisotropic equation where the strain solely depends on the applied stress and is independent of the stress direction. In the mechanics

of soils and materials, a common form of this anisotropic equation can be expressed as follows:

$$\varepsilon = a\sigma^2 + b\sigma + c \quad (1)$$

Where:

$\varepsilon$  is the deformation (strain) of the material.

$\sigma$  is the stress applied to the material.

A, B, and C are physical constants.

The least squares method is utilized for estimating the coefficients of a model in such a manner that the sum of the squares of the errors between actual data and the values predicted by the model is minimized. This method is applied to the outcomes of consolidation compression tests to obtain deformation indices at corresponding pressure levels. In the context of the anisotropic equation  $\varepsilon = a\sigma^2 + b\sigma + c$ , this approach facilitates the estimation of coefficients a, b, and c.

As previously discussed, this equation is represented by the tangent line equation of the stress-deformation relationship at a specified reference point  $P_{ref}$ . The equation of the tangent at the point  $P_{ref}$  is determined by using the derivative of the function at  $P_{ref}$ . The equation of the tangent line takes the form  $y = mx + c$ , where m represents the slope of the tangent, and c is the y-intercept at point  $P_{ref}$ , thereby determining the deformation coefficient  $\varepsilon$  corresponding to the stress  $P_{ref}$ .

First-order derivative of equations “Eq. (1)”  $\varepsilon' = 2a\sigma + b$ , Equation  $y = mx + c$  takes the form  $y = \varepsilon'(\varepsilon_0)(x - \varepsilon_0) + P_{ref}$ .

Determine  $E_{ode}^{ref}$  By calculating the slope through

a tangential line equation: 
$$E_{ode}^{ref} = \frac{y_j - y_i}{x_j - x_i} \quad (2)$$

## 4. RESULTS

### 4.1 Results of Geological Survey Experiments

Table 1. Results of mechanical and physical properties of the sample

Soil layer	Sample number	Depth (m)	W (%)	$\gamma_w$ (kg/m <sup>3</sup> )	$\gamma_d$ (kg/m <sup>3</sup> )	GS	e	n (%)	$\zeta$ (%)	LL (%)	PL (%)	PI (%)
Layer 1	UD1-1	1.5-2.0	23.7	1960	1580	2720	0.722	42	89	44	17	26
	UD1-2	7.0-7.5	15.5	2110	1830	2730	0.492	33	86	44	19	25
	UD2-1	1.5-2.0	21.1	2000	1650	2630	0.594	37	93	44	20	23
	UD2-2	7.0-7.5	19.4	2020	1690	2640	0.562	36	91	54	17	37
	UD3-1	1.5-2.0	20.6	2000	1660	2677	0.613	38	90	52	19	33
	UD4-1	7.0-7.5	21.9	2000	1640	2646	0.613	38	95	48	14	33
	UD5-1	1.5-2.0	19.2	2000	1680	2690	0.601	38	86	53	18	35
	UD5-2	7.0-7.5	15.1	2180	1890	2651	0.403	29	99	45	14	32
	UD6-1	1.5-2.0	21.1	1990	1640	2700	0.646	39	88	49	18	31
	UD6-2	7.0-7.5	19.9	1990	1660	2710	0.633	39	85	49	17	32
Average			19.8	2025	1692	2679	0.5879	37	90	48	17	31

Analysis of soil samples in the laboratory includes determining the particle composition and physicommechanical properties of the soil. The results are shown in Table 1 and Table 2. The results of testing the particle composition of the sample show that this soil layer (layer 1) is mixed clay with a depth of 7.5 m.

The average results of the mechanical properties of the sample at soil layer 1 are as follows: W% (19.8),  $\gamma_w$  (20.25),  $\gamma_d$  (16.92), GS (26.79),  $\varepsilon$  (0.5879), n % (37),  $\zeta$  % (90), LL % (48), PL % (17), PI % (31).

### 4.2 Determination of Modulus of Elasticity E' (E<sub>50</sub>) by Triaxial Compression Experiments CD

According to Table 2, it was found that the sample was numbered UD1-2 at a depth of 7.0 – 7.5 m, with a 3-axis CD compression experiment performed. Model UD1-2 was taken from borehole BH1 and had a length of 76 mm and a radius of 38 mm.

Where:  $\gamma_{sat} = (GS - 1) * 10 / (1 + e) + 10$  with  $\gamma_w$  is a constant, taking equal to 10 kN/m<sup>3</sup>, e is void ratio. The angle of internal friction  $\phi'$  is the angle between the tangent line with 3 mohr circles with the vertical axis và  $\Psi = \phi' - 30$  (if  $\phi' > 30$ ) or  $\Psi = 0$  (if  $\phi' \leq 30$ ).

The results of the CD 3-axis compression experiment of the UD1-2 sample are presented in Table 3, the parameters describe the soil in Table 4.

Deformation modulus value E' (kN/m<sup>2</sup>) determined by drainage consolidation compression experiment. The theory is based on the relationship between stress and strain (Fig. 2). Identify the E' corresponding to each pressure level  $\sigma_3$  (200, 300, and 400) (Table 3); draw a line representing the relationship E' corresponding to pressures 200, 300, and 400 (Table 5).

Table 2. Test results in quick, direct shear, triaxial compression (UU TEST)

Soil layer	Sample number	Depth	UU TEST		CU TEST			CD TEST	
			CUU	φUU	CCU	φCU	C'CU	φ'CU	CCD
Layer 1	UD1-1	1.5-2.0	97.3	01°46'					
	UD1-2	7.0-7.5						7.1	30°26'
	UD2-1	1.5-2.0						12.5	27°32'
	UD2-2	7.0-7.5			24.6	16°47'	15.7	27°41'	
	UD3-1	1.5-2.0			29.7	16°30'	16.7	25°43'	
	UD4-1	7.0-7.5							
	UD5-1	1.5-2.0							
	UD5-2	7.0-7.5	174.6	04°32'					
	UD6-1	1.5-2.0							
UD6-2	7.0-7.5								
Average			136.0	03°09'					

Table 3. Consolidated drained triaxial compression test

Specimen No	1	2	3	
Strain %	5.58	4.44	6.5	
Deviator stress kg/cm <sup>2</sup>	4.334	6.455	8.442	
σ <sub>3</sub> kg/cm <sup>2</sup>	2	3	4	
σ <sub>1</sub> kg/cm <sup>2</sup>	6.334	9.455	12.44	
Failure condition	Volume strain %	0.779	1.044	1.955
	The angle of internal friction			30°26'
Shear strength	Cohesive strength kG/cm <sup>2</sup>			0.071
E <sub>50</sub> kG/cm <sup>2</sup>		411.4	660.1	821.8
		1	3	8
Strain at failure 50% deviatoric stress		0.53	0.49	0.51

Horizontal stress of soil samples in their natural state (L=7.5 m)  $P_{ref} = (L/2) * \gamma_{unsat} * (1 - \sin \phi')$ . So  $P_{ref} = 38$ . Define  $E'$  ( $E_{50}$ ) corresponds to  $P_{ref} = 38$  by substituting the  $P_{ref}$  value into the Relationship equation between  $E'$  and  $\sigma_3$ :  $y = 623.67x + 4918.4 = 28313$  (kN/m<sup>2</sup>) (Fig. 2). Poisson's ratio:  $\nu' = 0.25 + 0.00225 * PI$  (where: PI is the Plasticity Index (%)). From experimental results  $\nu' = 0.25 + 0.00225 * 31$  (Table 1).

The permeability coefficient in this study was considered from sample UD6-2, the sample tested at the bottom depth of layer 1.

With value  $P_{ref} = 38$ , within the pressure range P: 0.25 - 0.5, and the void ratio 0.608 is close to the initial void ratio  $\epsilon_0$ , the permeability coefficient can be selected to approximate the permeability coefficient value  $K_z = 1.66 E^{-08}$  (Table 8). Coefficient  $K_x = K_y = (2 - 3)K_z$ , in this study choose  $K_x = K_y = 2.5K_z$ ;

Table 4 provides a detailed summary of the parameters and units used in the soil model. These values, derived from experimental data, form the basis for analyzing soil behavior under various

conditions, ensuring the accuracy and reliability of the model.

Table 4. The value of parameters and units of the soil model

No.	Content	Unit	Value
1	Class name		Layer 1
2	Length L(m)	m	7.5
3	Type of behavior of the soil		UD
4	$\gamma_{unsat}$	(kN/m <sup>3</sup> )	20.25
5	$\gamma_{sat}$	(kN/m <sup>3</sup> )	20.57
6	Void ratio (e)		0.5879
			28313
			(according to the results of the CD 3-axis compression experiment)
7	Deformation modulus (E')	(kN/m <sup>2</sup> )	
8	Poisson's ratio ( $\nu'$ )		0.32
9	Adhesive force ( $c'$ )	(kN/m <sup>2</sup> )	7.1
10	Angle of internal friction ( $\phi'$ )	Degree	30.4
11	$\Psi$	Degree	0.4
12	$K_z$	m/day	1.43E-05
13	$K_x = K_y$	m/day	3.59E-05

This meticulous approach ensures that the model operates with a high degree of accuracy and reliability, making it a reliable tool for scientific investigation into soil dynamics.

Table 5. The value E' (E<sub>50</sub>) corresponds to the sad pressures

σ <sub>3</sub> (kN/m <sup>2</sup> )	σ <sub>1</sub> max	E50 (σ <sub>1</sub> /2)	ε (%) The deformation corresponds to E50	E' (E <sub>50</sub> /ε) (kN/m <sup>2</sup> )
200	633	316.5	0.00245	129184
300	945.5	472.75	0.00245	192959
400	1244.2	622.1	0.00245	253918

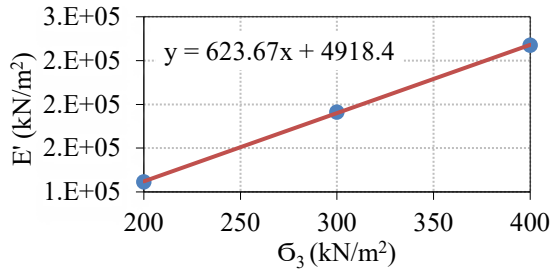


Fig. 2 Relationship between E' and σ<sub>3</sub>

The results of the compression experiment are described in Table 6.

Table 6. A part of compression test results

Sample number	Void ratio at load P (kN/m <sup>2</sup> )					
	25	50	100	200	400	600
UD6-2	0.617	0.608	0.592	0.570	0.544	0.53
Average	0.641	0.630	0.612	0.585	0.549	0.53

Table 7. BH6-2 sample compression test results

BH6-2				
Cc (kN/m <sup>2</sup> )	W (%)	γ <sub>w</sub> (g/cm <sup>3</sup> )	γ <sub>d</sub> (g/cm <sup>3</sup> )	G <sub>s</sub>
0.097	19.9	1.99	1.66	2.71
ε <sub>0</sub>	n (%)	S (%)	P <sub>c</sub> (kg/cm <sup>2</sup> )	Cr (kN/m <sup>2</sup> )
0.633	38.75	85.26	0.85	0.017

Table 8. The result of P, ε, a<sub>v</sub>, E, C<sub>v</sub>, and K values when changing the P value.

P	e	a <sub>v</sub>	E	C <sub>v</sub>	K
0-0.25	0.617	0.046	35.118	2.61E-04	1.96E-08
0.25-0.5	0.608	0.038	43.076	2.74E-04	1.66 E-08
0.5-1.0	0.592	0.032	50.255	2.82E-04	1.44 E-08
1.0-2.0	0.570	0.022	73.600	2.57E-04	8.78 E-08
2.0-4.0	0.544	0.013	117.670	1.93E-04	4.00 E-09
4.0-6.0	0.527	0.009	180.114	1.61E-04	2.12 E-09

### 4.3 Determination of the Modulus of Elasticity E' (E<sub>50</sub>) by Compression Experiment

The relationship between E' (E<sub>50</sub>) and E<sub>oed</sub> is according to the formula: E' (E<sub>50</sub>) = 1.25E<sub>oed</sub>. From the results of the 1-axis compression experiment (Table 7, Table 8). Draw the relationship line between stress and strain. Settlement value  $s = (\epsilon_0 - e) * h_0 / (1 + e_0)$  where e<sub>0</sub> is the initial void ratio, e void ratio corresponds to each pressure level, and h<sub>0</sub> is the initial size of the experimental sample (h<sub>0</sub> = 2 cm). Table 9 presents the numerical values of settlement (s) and deformation (ε).

First-order derivative of equations  $\epsilon' = 2a\sigma + b$ , Equation  $y = mx + c$  takes the form  $y = \epsilon'(\epsilon_0)(x - \epsilon_0) + P_{ref}$ . Calculate the value x at the point where P<sub>ref</sub> = 38 has the equation  $38 = 14.162x^2 - 2.9368x + 10.28$  (Fig. 3). Solve the system of equations for the solution  $x = 1.506$ . The tangent line equation of the form  $y = mx + c$  has the form  $y = \epsilon'(\epsilon_0)(x - \epsilon_0) + P_{ref}$  so  $y = 39.719x - 21.817$  (Fig.3).

According to the definition, E<sub>oed</sub> has a value of 3971.914 (kN/m<sup>2</sup>) (Fig. 4), so E' (E<sub>50</sub>) = 1.25 E<sub>oed</sub> = 4965 (kN/m<sup>2</sup>).

Table 9. Settlement values s and ε corresponding to each pressure level

P (kN/m <sup>2</sup> )	Settlement (S)		Deformation (ε) (%)
	Void ratio (e)	(cm)	
0	0.633	0	0
25	0.617	0.02	0.979792
50	0.608	0.031	1.530925
100	0.592	0.05	2.510716
200	0.57	0.077	3.85793
400	0.544	0.109	5.450092
600	0.527	0.13	6.491121

From stress and strain values, make an inequality line equation of the form  $\epsilon = a\sigma^2 + b\sigma + c$

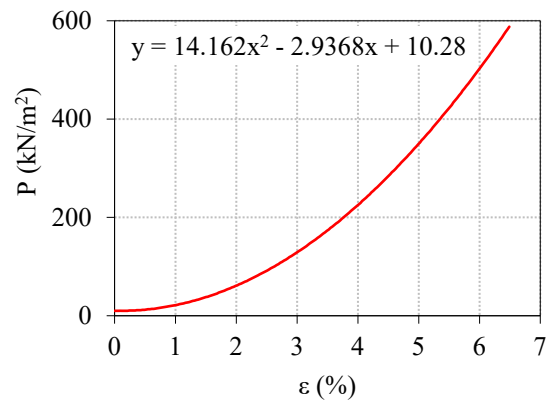


Fig. 3 Relationship between P and ε

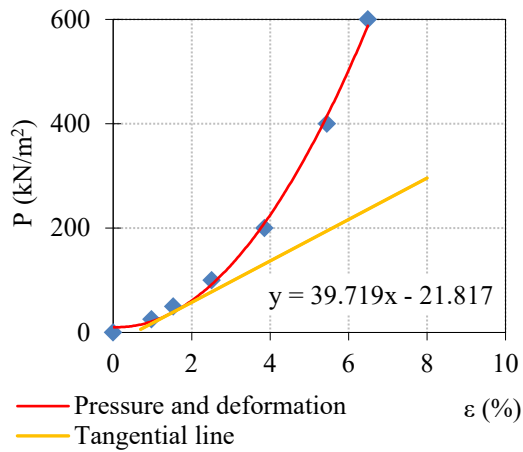


Fig. 4 Tangential line at  $P_{ref}$  point

**4.4 Determination of the Modulus of Elasticity  $E'$  ( $E_{50}$ ) by Standard Penetration Test (SPT)**

According to Terzaghi, Peck, and Meyerthof, the relationship between the angle of internal friction and the standard penetration resistance is as follows:

$$\varphi = \sqrt{12 N_{spt}} + a \tag{3}$$

Where:  $a$  is the coefficient, taking the value between 15 and 20

The strain modulus  $E$  is calculated in Tassios, Anagnostopoulos:

$$E = \frac{a + c(N_{spt} + 6)}{10} \tag{4}$$

Where:

$a$  is the coefficient taken equal to 40 when the  $N_{spt} > 15$ ; get zero when  $N_{spt} < 15$

$c$  is that the coefficient is taken that depends on the type of soil

The SPT closure result of layer 1 soil layer has  $N_{spt} = 10$ ; for clay soil,  $c = 3$  should be selected. Apply the defined formula  $E'$  ( $E_{50}$ ) = 4800 kN/m<sup>2</sup> (Eq. (4)).

**5. DISCUSSION**

In order to find more efficient and cost-effective ways of determining the elastic modulus ( $E'$ ) of soil, which is a crucial parameter in geotechnical engineering, it is necessary to reassess traditional testing techniques. This research pivots toward the innovative application of derivative-based calculations and one-dimensional consolidation test outcomes as viable alternatives to resource-intensive triaxial CD testing. Given the significant time and financial investments required by triaxial CD tests, our study introduces a derivative approach to estimate the soil's  $E'$  at a specific reference stress point ( $P_{ref}$ ), by deriving a tangent equation that mirrors the compression behavior of soil with high fidelity. This approach not only reduces the intricacy associated

with traditional tests but also provides a fast and cost-effective means of evaluating soil rigidity. Furthermore, the study explores the efficacy of utilizing results from one-dimensional consolidation tests to ascertain  $E'$ , providing a compelling argument against the sole reliance on time-consuming and costly triaxial CD tests. By analyzing soil behavior under varied loading stages during consolidation, we were able to extrapolate  $E'$  values that closely match those obtained from triaxial tests. This approach not only confirms the feasibility of using consolidation test data to assess stiffness but also greatly reduces the reliance on extensive triaxial testing. As a result, it suggests a more cost-effective and timely method for geotechnical assessments. The  $E'$  value is presented in Table 10.

Based on the results from the consolidated drained triaxial compression (CD) test, the elastic modulus ( $E'$ ) was found to be 28313 kN/m<sup>2</sup> at a depth of 3.75 m. The calculation of  $E'$  from the consolidation compression results showed a decrease in value due to the absence of overburden pressure, corresponding to a zero-depth modulus recovery. When evaluating  $E'$  from Standard Penetration Test (SPT) results in an unsaturated state, high accuracy was achievable primarily in sandy soils. The analysis of the void ratio ( $e = 0.588$ ) and related indices indicates significant consolidation of this soil layer. Substituting a  $P_{ref}$  value of 0 (representing natural ground pressure at surface level) resulted in an  $E'$  of 4918 kN/m<sup>2</sup>. This number closely aligns with the  $E'$  values derived from both the SPT and the consolidation compression tests (Table 10).

Table 10. Compare the  $E'$  strain modulus results with those from previous research

	CD 3-axis compression experiment	Test Consolidation	Standard Penetration Test (Tassios, Anagnostopoulos)
$E'$ kN/m <sup>2</sup>	28313	4965	4800

Considering the increase in the deformation modulus with depth, the consolidation compression test results indicated an  $E'$  of 4965 kN/m<sup>2</sup> at the surface level, which is close to the results of Tassios and Anagnostopoulos (4800 kN/m<sup>2</sup>), the coefficient of increase in depth per meter is 6238 kN/m<sup>2</sup>. Consequently,  $E'$  at the depth corresponding to  $P_{ref}$  (3.75 m) was calculated to be 26799 kN/m<sup>2</sup>, which is close to the results of the CD 3-axis compression experiment (28313 kN/m<sup>2</sup>). This observation underscores that  $E'$  estimated from consolidation compression tests correlates well with those from triaxial CD tests.  $E'$  values derived from in-situ SPT results are congruent with those obtained from consolidation compression tests at the natural ground level ( $h = 0.0$  m), highlighting the consistency across

different testing methodologies.

Our research demonstrates that derivative calculations and consolidation test results are effective methods for determining the soil's  $E'$  in geotechnical engineering, thus contributing to the field's advancement. This revelation is set to streamline, economize, and enhance the efficiency of soil mechanics analysis, allowing for accurate soil behavior assessments without the extensive costs and time traditionally associated with triaxial CD testing. Future studies are encouraged to expand the application of these methods across diverse soil types and conditions, potentially revolutionizing soil mechanics analysis and its integration into engineering projects.

## 6. CONCLUSIONS

This research focuses on the effectiveness and feasibility of using derivative-based approaches and consolidation test outcomes to determine the elastic modulus ( $E'$ ) of soil, a crucial factor in geotechnical engineering. The study advocates for alternative approaches that streamline the process and maintain accuracy in determining soil's mechanical properties. This study presents a new method for accurately estimating the value of  $E'$  by utilizing the derivative of the stress and deformation relationship. By applying derivative calculations at a specific reference stress point ( $P_{ref}$ ), we derived a tangent equation that closely approximates the soil's behavior under compression. This method not only simplifies the complexity involved in traditional testing methods but also provides a rapid assessment tool for evaluating soil stiffness with minimal resource expenditure.

Moreover, utilizing findings from one-dimensional consolidation tests to calculate  $E'$  offers a convincing substitute for the laborious and expensive triaxial CD tests. Through careful analysis of consolidation behavior under different load stages, we successfully extrapolated  $E'$  values that align closely with those obtained from triaxial tests. This approach not only confirms the effectiveness of using consolidation test data to evaluate stiffness, but also greatly reduces the requirement for extensive triaxial testing. It provides a cost-effective and time-efficient method for geotechnical assessments.

In essence, this study advances the field of geotechnical engineering by demonstrating that derivative calculations and one-dimensional consolidation test results can effectively determine soil's elastic modulus ( $E'$ ). This discovery facilitates the development of a more efficient and cost-effective method for analyzing soil mechanics. It allows practitioners to accurately evaluate soil behavior without the excessive expenses and time constraints typically associated with triaxial CD testing. Future research should further explore the application of

these methods across a broader range of soil types and conditions, potentially revolutionizing soil mechanics analysis and its application in engineering projects.

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